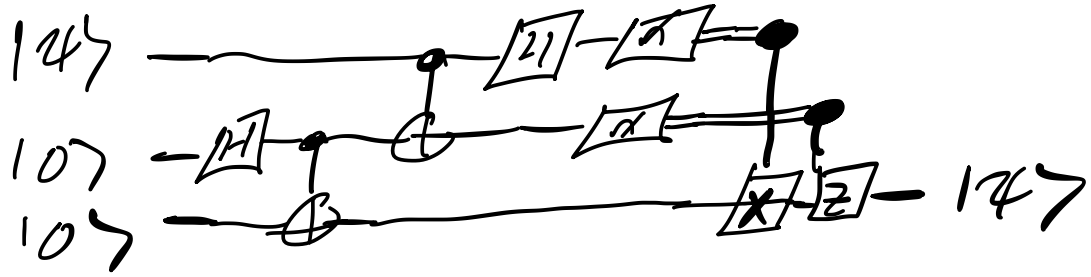
1. Qubit routing motivation (Qiskit)
2. Idea: Teleport!
3. Routing time + bounds
4. Three examples where TELE beats SWAP of $G + \pi$
5. One example of G where TELE beats SWAP for any π
6. Behavior for any G, any π
   - A few classes of G (any π)

1 Qiskit SWAP Example

② Teleportation



$$|\psi\rangle|00\rangle + |\psi\rangle|11\rangle$$

$$= \alpha|000\rangle + \beta|100\rangle + \alpha|011\rangle + \beta|111\rangle$$

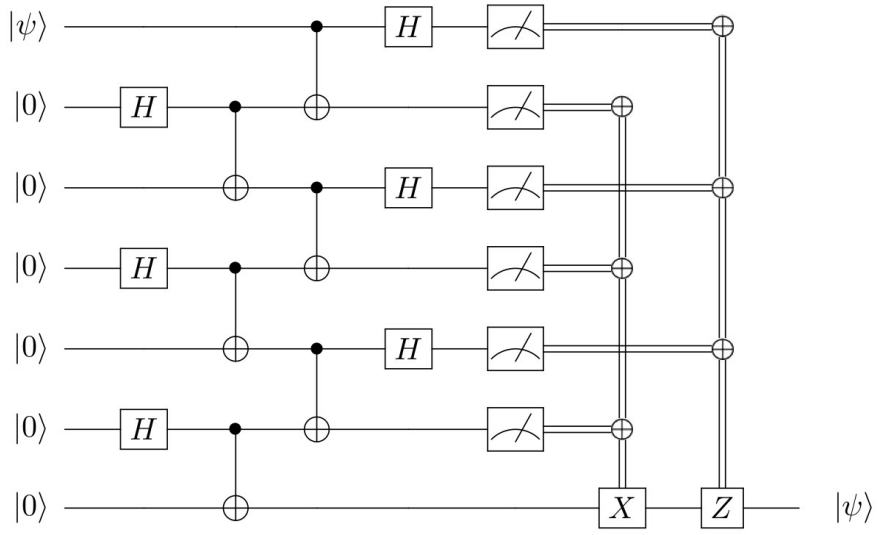
$$\mapsto \alpha|000\rangle + \beta|111\rangle + \alpha|011\rangle + \beta|100\rangle$$

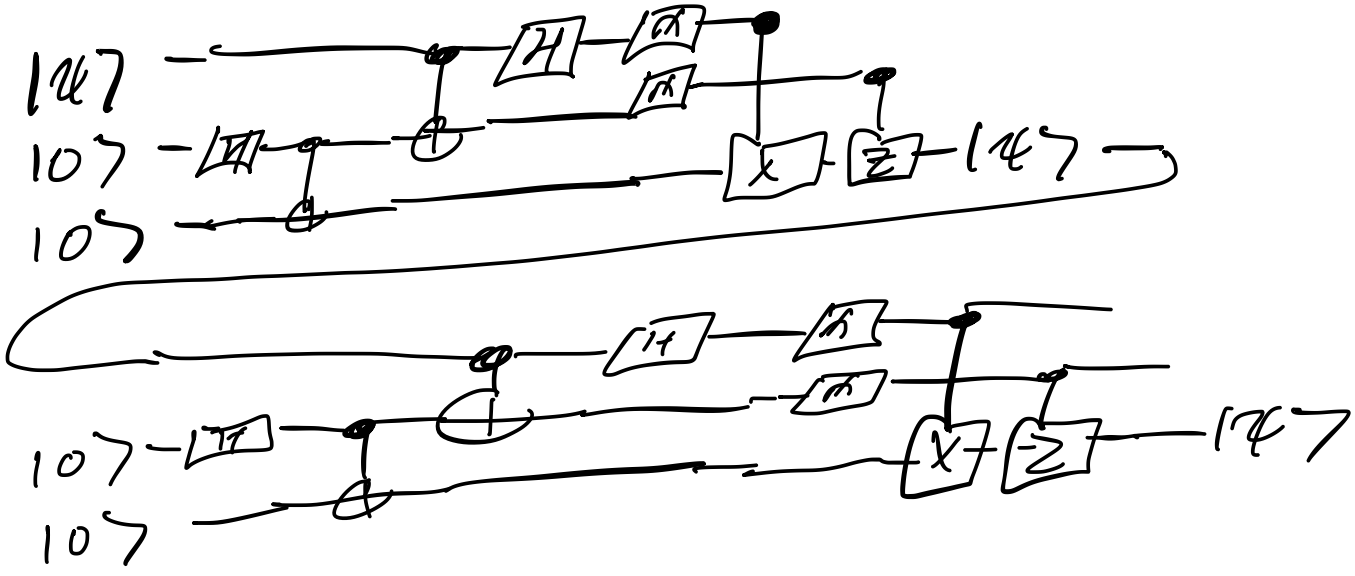
$$\mapsto \alpha|000\rangle + \alpha|110\rangle + \beta|010\rangle - \beta|111\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|001\rangle - \beta|110\rangle$$

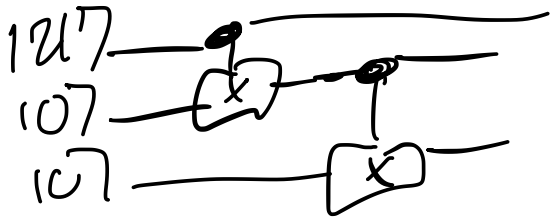
$$= |00\rangle[\alpha|0\rangle + \beta|1\rangle] + |01\rangle[\beta|0\rangle + \alpha|1\rangle] + |10\rangle[\alpha|0\rangle - \beta|1\rangle] + |11\rangle[-\beta|0\rangle + \alpha|1\rangle]$$

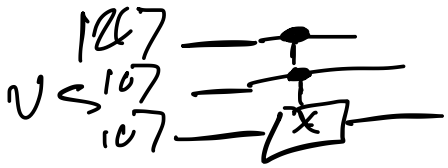
$$= |00\rangle|\psi\rangle + |01\rangle X|\psi\rangle + |10\rangle Z|\psi\rangle + |11\rangle ZX|\psi\rangle$$

Long-range teleportation in constant depth









$\alpha|000\rangle + \beta|100\rangle$

$\alpha|000\rangle + \beta|111\rangle$

③ Routing time + bounds

$G = (V, E)$, permutation π

$rt(G, \pi)$:= Min. circuit depth to perform π on G

Worst case routing time

$$rt(G) := \max_{\pi \in S_{|V|}} rt(G, \pi)$$

Where S_N is the symmetric group (permutations of N elements)

Lower bounds on routing time

Let $X \subseteq V$.

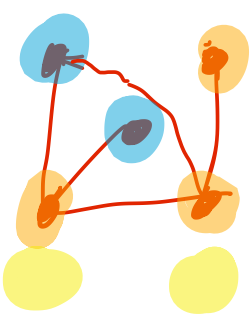
$\bar{X} := V - X$ (Complement)

$$\delta X := \{ v \in \bar{X} : \exists u \in X \text{ s.t. } (u, v) \in E \}$$

(Vertex boundary)

Ex

G



$\bar{X}$

δX

(Note: $\delta X \subseteq \bar{X}$)

Define the **Vertex expansion** of G as

$$c(G) := \min_{X \subseteq V} \frac{|\delta X|}{\min(|X|, |\bar{X}|)}$$

- Since $\delta X \subseteq \bar{X}$, $c(G) \leq 1$.
- for connected graphs, $c(G) \in \left[\frac{2}{N}, 1\right]$

Theorem: Min routing time

$$rt_{SWAP} \geq rt_{tele} \geq \frac{2}{c(G)} - 1$$

- for SWAP routing,

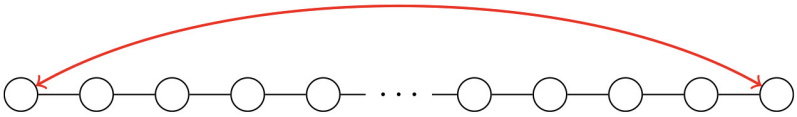
$$rt_{SWAP} \geq diam(G)$$

Upper bounds on routing time

$$rt_{SWAP} = rt_{tele} = O(|V|)$$

(1) Three examples of $G + \pi$

Ex 1



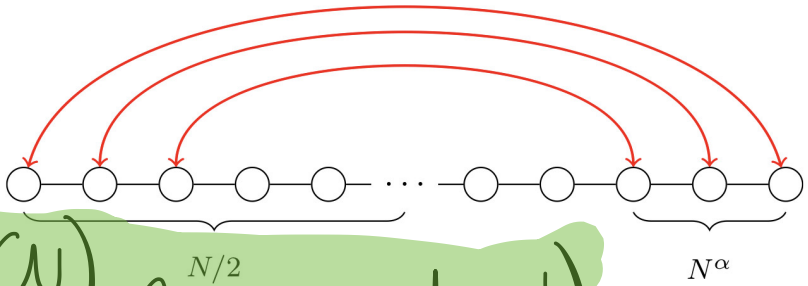
N nodes

$$rt_{SWAP}(G,\pi) = \Omega(N)$$

$$rt_{SWAP}(G,\pi) = O(1)$$

$$adv(G,\pi) := \frac{rt_{SWAP}(G,\pi)}{rt_{TELE}(G,\pi)} = O(N)$$

Ex 2



$$rt_{SWAP}(G,\pi) = \Theta(N)$$ (diameter bound)

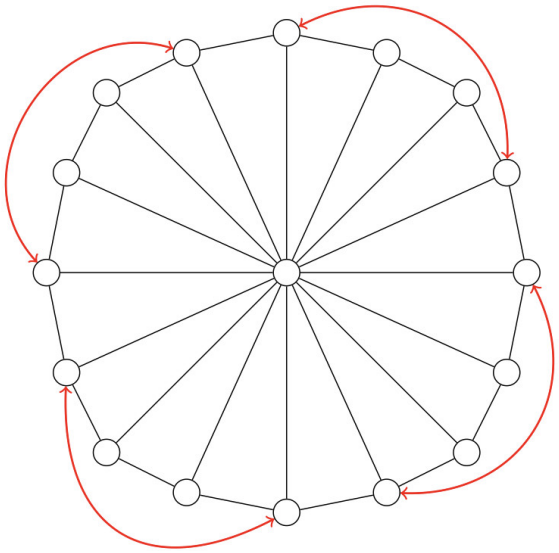
$$rt_{TELE}(G,\pi) = O(N^\alpha)$$

$\Rightarrow$ polynomial advantage $adv(G,\pi) = O(N^{1-\alpha})$

Q: Are we just getting advantages by virtue of the large diameter (N) of these Graphs?

Ex 3

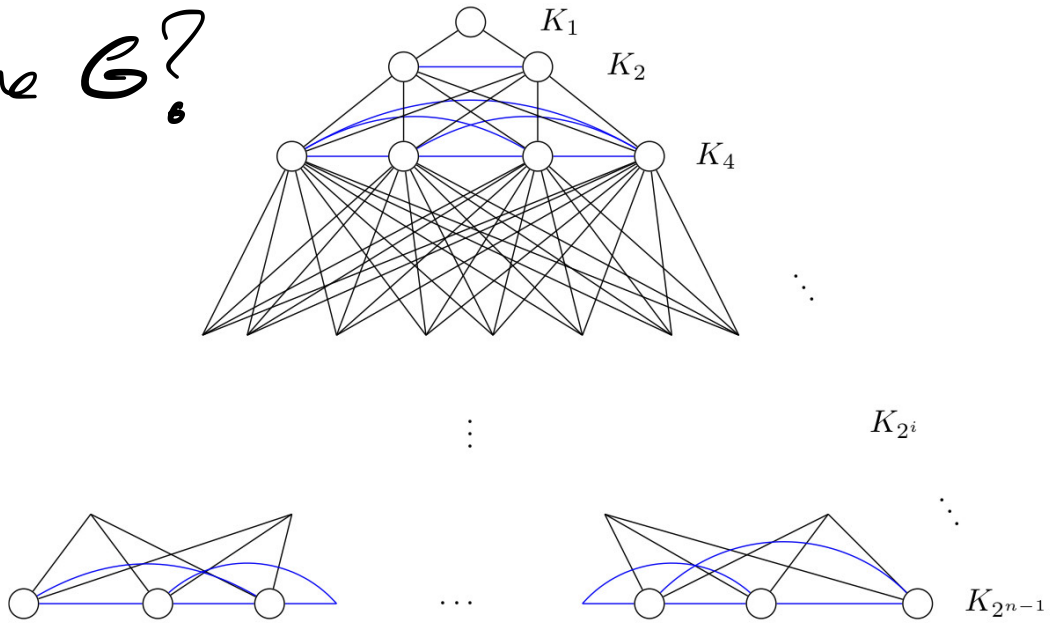
N+1 nodes

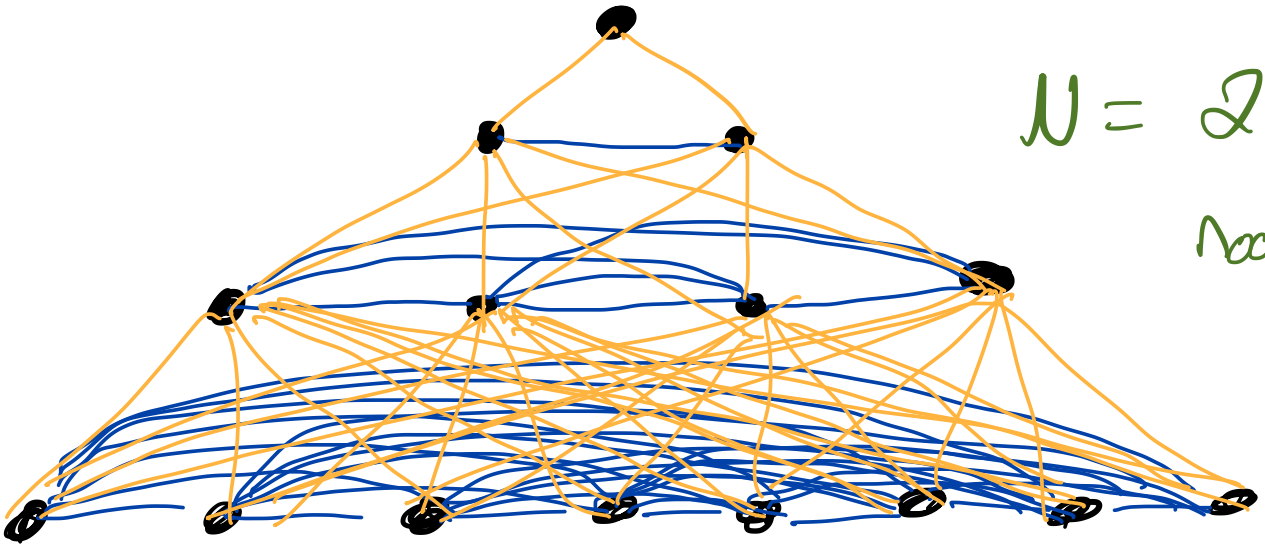


diam(G) = 2

(5) One example of G (any TC)

What's the G?



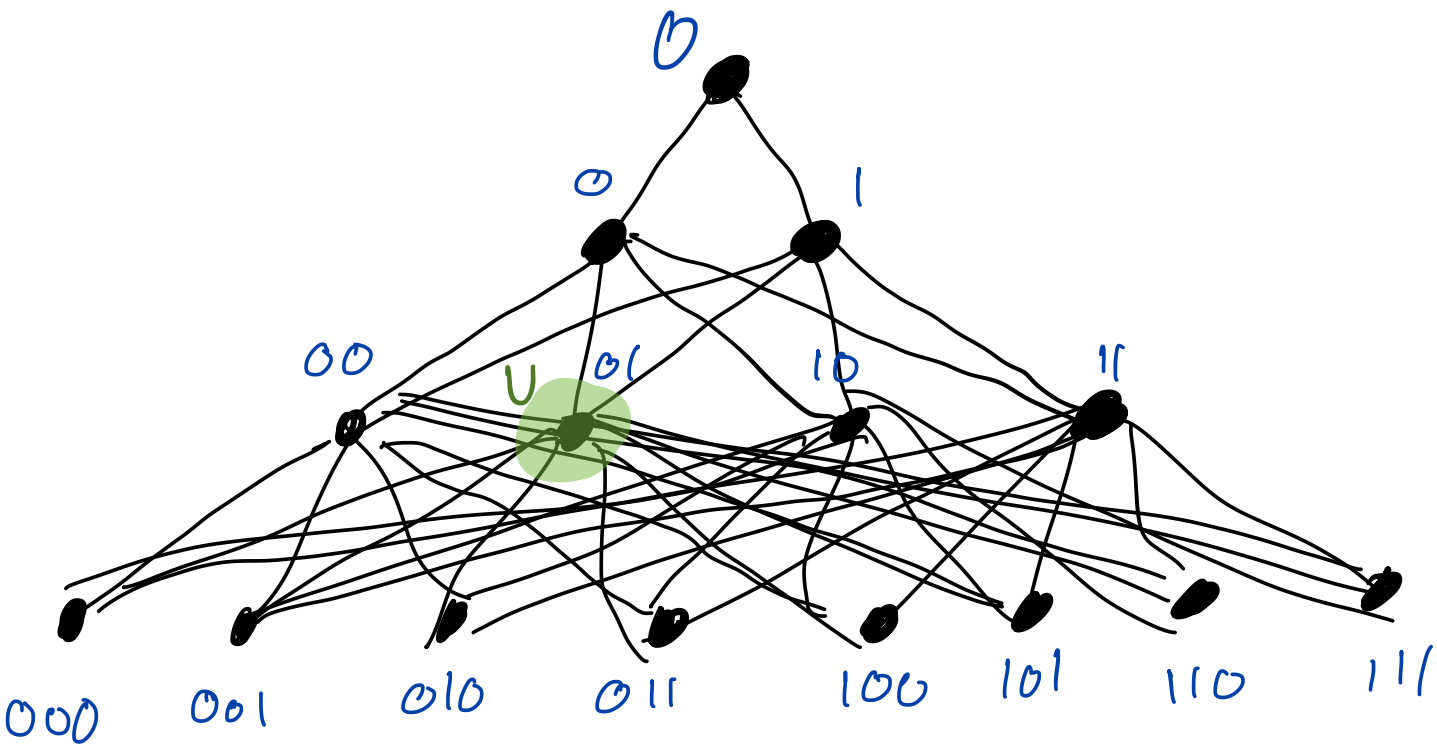


$N = 2^n - 1$ nodes

Thm: $rt_{SWAP}^{(G)} = \Omega(\log N)$

Thm: $rt_{TELE}^{(G)} = O(1)$

Teleportation paths (Canonical paths)



⑥ General behavior for any G and any π

$$adv^* = O\left(\sqrt{N \log N}^{-1}\right)$$

$$adv^* := \max_G adv(G)$$

$$adv(\text{planar grids}) = O(1)$$