

The Deutsch-Jozsa algorithm

Consider functions f which input one bit (0 or 1) and output one bit (0 or 1). How many different functions f are there?

Draw graphs for all possible functions f from one bit to one bit.

Which functions above are constant? Which are balanced?

Activity: Play the DJ game at your table. Record how many questions you need to ask for each round. Make sure everyone gets a chance to be the “oracle.”

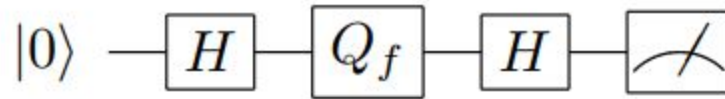
Define the parity as $f(0) + f(1)$ where addition is carried out modulo two.

(That is, $0 + 0 = 0$, $0 + 1 = 1$, $1 + 0 = 1$, $1 + 1 = 0$.)

How can determining the parity tell us if the function f is constant or balanced?

Suppose now we have access to a quantum computer. What is the quantum query complexity of the DJ problem/game?

Here's the quantum circuit diagram for the DJ problem. Label each part of the diagram and answer questions as we go through it.



What is the state of our qubit after the first Hadamard gate?

What is the state of our qubit after the query Q_f ?

What is the state of our qubit after the second Hadamard gate?

If f is constant ($f(0) = f(1)$), what will our detector always measure?

If f is balanced ($f(0) \neq f(1)$), what will our detector always measure?

Conclude: How many questions did it take to solve the DJ problem in the quantum case?