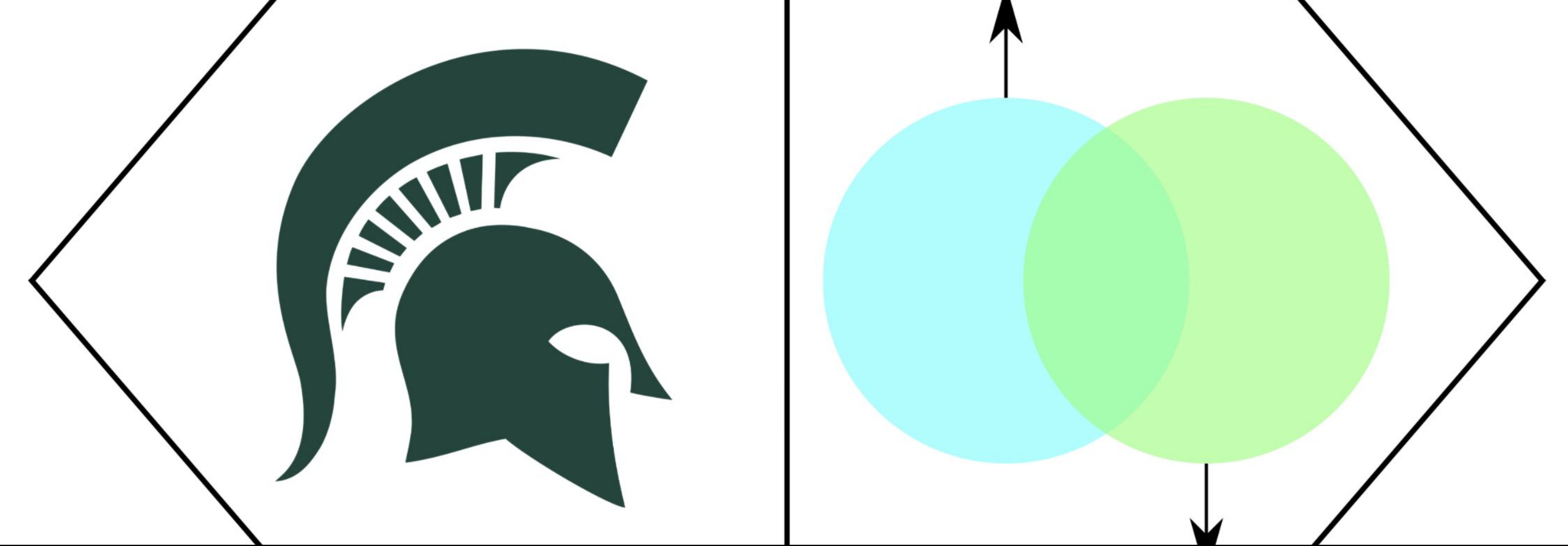
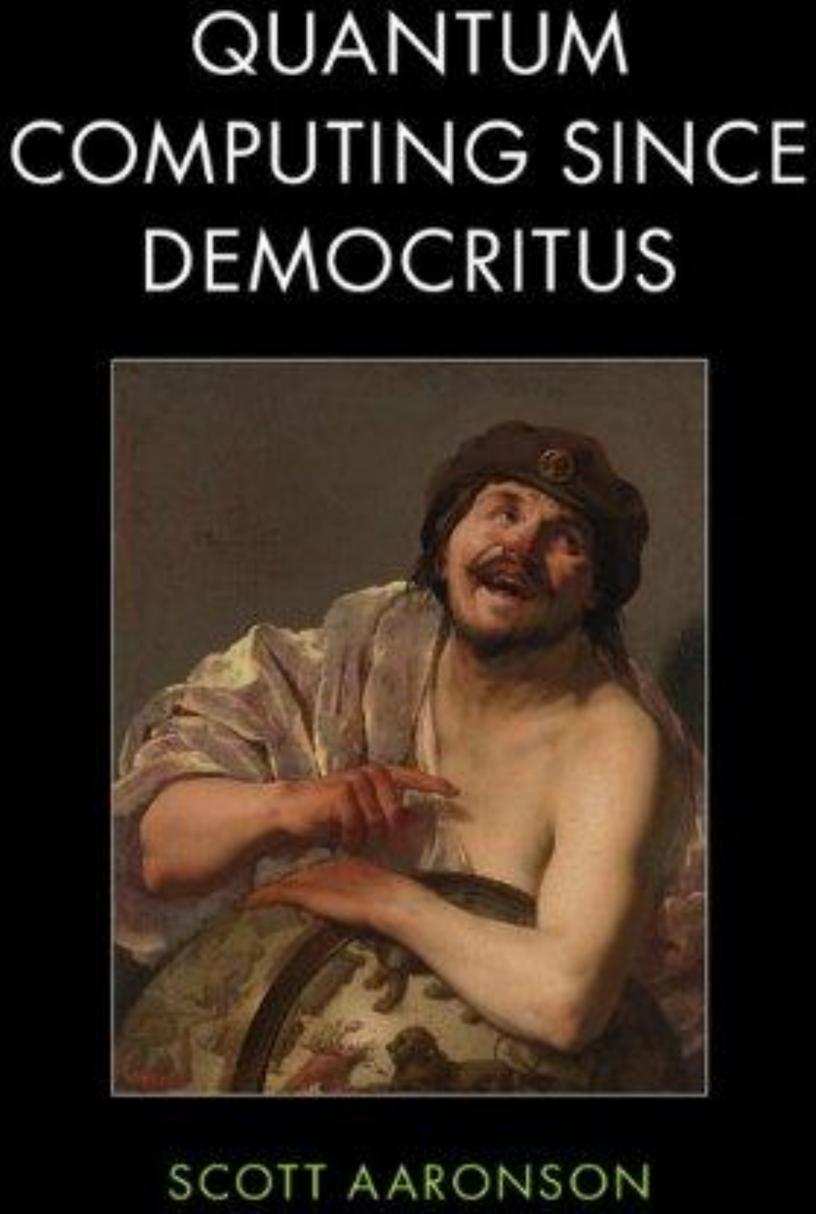


Why quantum uses the 2-norm

A foray in counterfactual reasoning



Inspiration for this talk

- Great book by well-regarded, hilarious complexity theorist Scott Aaronson
- Not quite a textbook, not quite popular science
- Other than Ryan’s talk on the NP-hardness of Mario, taught me everything I know about theoretical computer science

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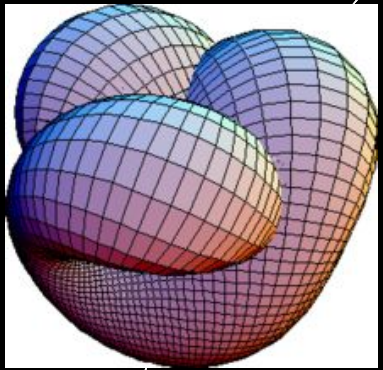
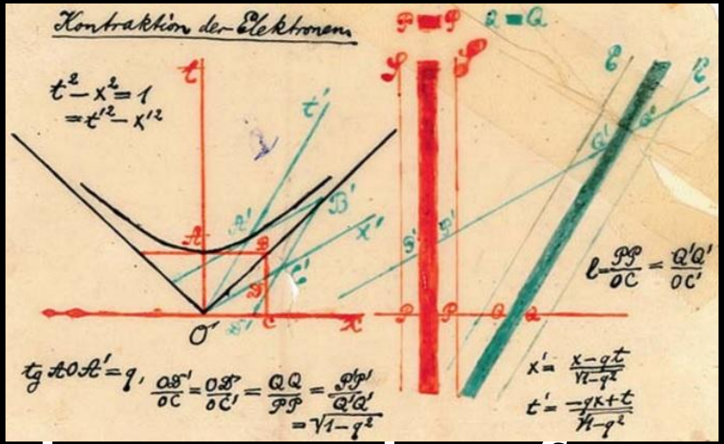
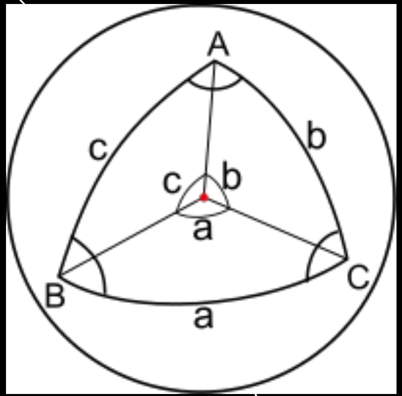






Could the math of quantum mechanics have been figured out before the experiments of the early 20th century?

My view (and Scott Aaronson’s):

YES!

What is the mathematical core of quantum?

- It's not ∞-dimensional vector spaces, differential equations, perturbation theory
- Maybe it is matrices, noncommuting observables? But I think this is secondary
- It definitely is probability theory with a twist.

$$\sum_j p_j = 1 \quad \longrightarrow \quad \sum_j \left|\psi_j\right|^2 = 1$$

Generalizing probability theory (GPT)

- Why not have negative probabilities?

- Okay fine… likelihood must be positive *in the end*

- $p_j = |v_j|$
- $p_j = v_j^2$
- …
- $p_j = |v_j|^p$

Any p-norm will work

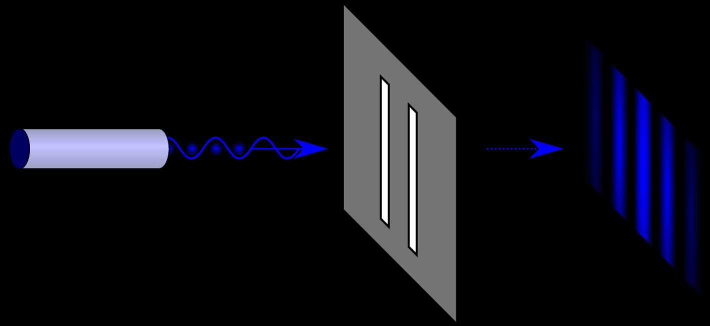
$$\sum_j |v_j|^p = 1$$

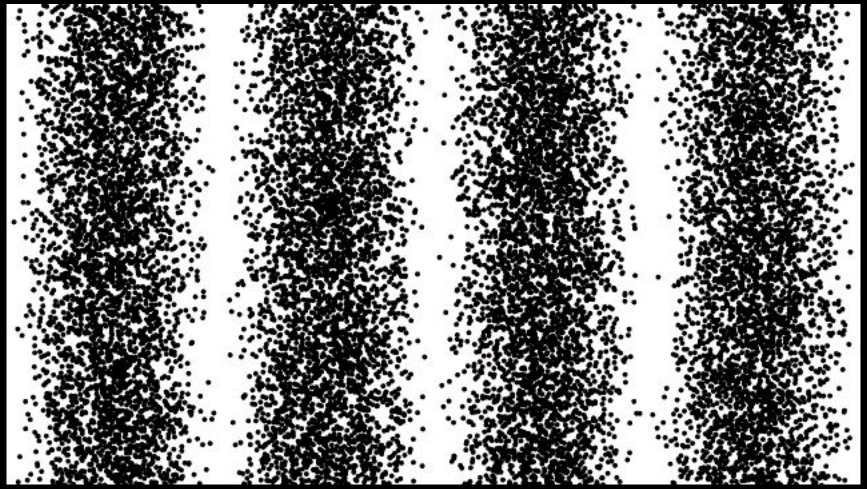
for $p \geq 0$

- It comes from the intermediate steps: how do the vectors transform?
- In classical probability, these are **stochastic processes** (e.g. Markov chains, Brownian motion,…)
- In quantum, these transformations are different $\Longrightarrow$ Quantum Computing

Fast forward to the 20th-century.

Nature is probabilistic, but we need amplitudes which can *interfere*.





Double-slit experiment, Wikipedia

Of all the generalizations of probability, why does nature choose one based on the 2-norm?

?

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What is she talking about?

-

1. "Wavelike" means our transformation of $v = (v_1, v_2, \ldots, v_n)$, Λ, should be linear

$$\Lambda(c_1 v_1 + c_2 v_2) = c_1 \Lambda v_1 + c_2 \Lambda v_2$$

Yay, Λ is a matrix!

2. "Interference" means v_j can be negative (or complex! Quaternion?)

Awesome fact:

Let Λ be a complex $n \times n$ matrix which preserves the p-norm for any vector $v \in \mathbb{C}^n$ and $p \geq 0$. If $p \neq 2$, then Λ is nothing more than a phased permutation matrix.*

I.e. the only thing Λ is allowed to do is swap entries and maybe add a phase to each spot. Lame!

If we want interesting transformations, we need the 2-norm.

*Stated in Aaronson's book without proof

Jacob's fact:

Let Λ be a real $n \times n$ matrix which preserves the p-norm for any vector $v \in \mathbb{R}^n$ and $p > 0$ and even. If $p \neq 2$, then Λ is nothing more than a signed permutation matrix.

This still rules out many possibilities!

(I've also proved for $p = 1$ separately. Generalizing my proof to complex Λ isn't hard.)

Intuition: as p grows, so does the # of constraints on Λ

To the blackboard!

Conclusion

- It took harsh experimental reality to lead humans to develop the mathematics of quantum mechanics.
- *That does not mean humans could never have done it beforehand*
- It *is* possible, and worthwhile, to consider why quantum theory is the way it is and why many other possibilities are ruled out.

Quantum is fascinating, but not all of it is mysterious.