

Classical addition

$a = 7$

$b = 4$

$$\begin{array}{r}
 1 \\
 7 \\
 + 4 \\
 \hline
 11
 \end{array}$$

Sum
Carry

Q: How do computers represent #s?

$$\begin{array}{r}
 a = 7 = \begin{array}{ccc} 1 & 1 & 1 \\ \hline 1 & 0 & 0 \end{array} = 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 \\
 b = 4 = \begin{array}{ccc} 1 & 0 & 0 \\ \hline 1 & 0 & 0 \end{array}
 \end{array}$$

Full adder
Input: 3 bits
Output: Sum, Carry

$$\begin{array}{r}
 1 \\
 111 \\
 + 100 \\
 \hline
 1011
 \end{array}$$

Half adder
Input: 2 bits
Output: Sum, Carry

Q: How do computers perform op's?

Operations

NOT

| a | NOT a |
|---|-------|
| 0 | 1 |
| 1 | 0 |

XOR

| a | b | a XOR b |
|---|---|---------|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

AND

| a | b | a AND b |
|---|---|---------|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

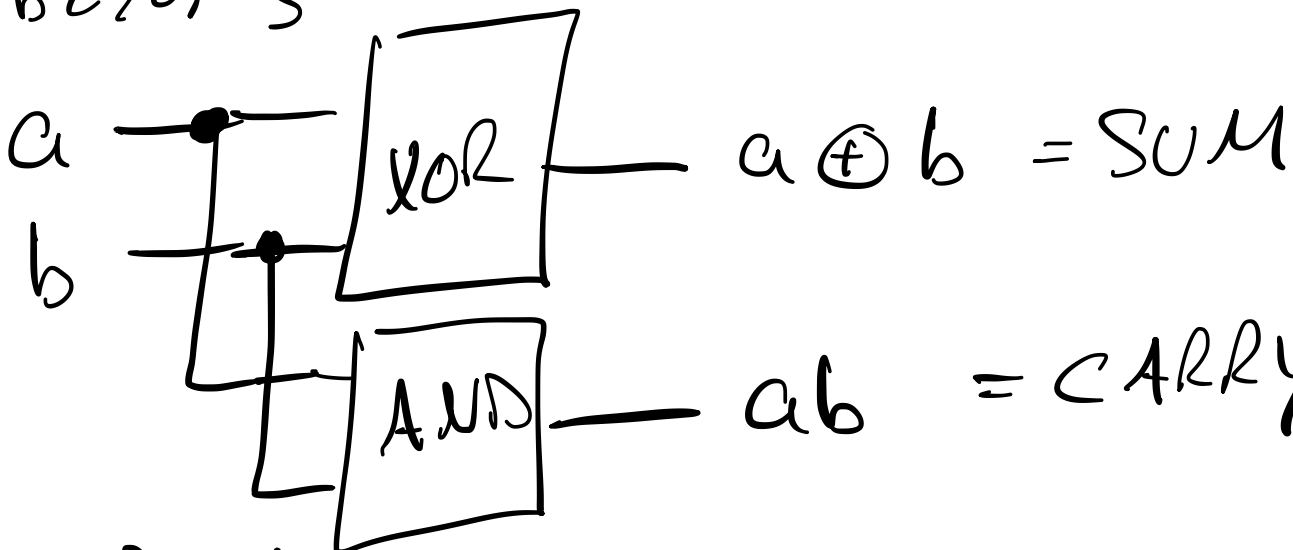
SUM

CARRY

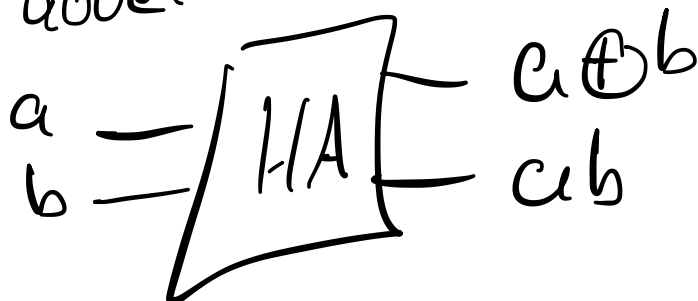
AND!

XOR!

$a, b \in \{0, 1\}$



Half adder



Full adder

| a | b | C _{in} | SUM | CARRY |
|---|---|-----------------|-----|-------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$$\text{SUM} = a \oplus b \oplus C_{in}$$

$$\text{CARRY} = ab \oplus C_{in} (a \oplus b)$$

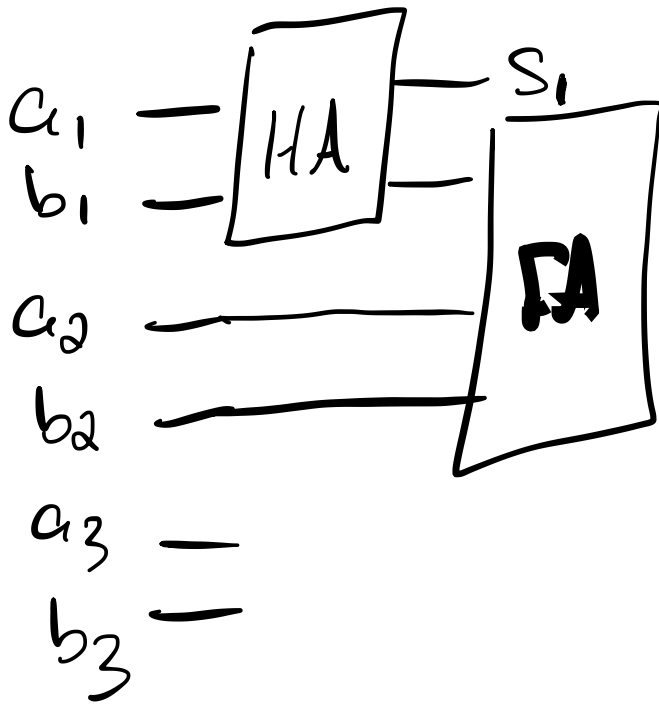
Ripple Carry adder

$$a = a_3 a_2 a_1$$

$$b = b_3 b_2 b_1$$

$$a_i \in \{0, 1\}$$

$$b_i \in \{0, 1\}$$



$$S = S_4 S_3 S_2 S_1$$