QuIC

Intro to quantum information + computation

through

Classical + quantum addition

Classical addition

$a = 7$
$b = 4$

$$\begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\ 4 \\ \hline 1\ 1 \end{array}$$

Sum

Carry

Q: How do computers represent #s?

$2^2\ 2^1\ 2^0$

$a = 7 = 1\ 1\ 1 = 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

$b = 4 = 1\ 0\ 0$

$$\begin{array}{r} {\scriptstyle 1} \\ 1\ 1\ 1 \\ +\ 1\ 0\ 0 \\ \hline 1\ 0\ 1\ 1 \end{array}$$

Full adder
Input: 3 bits
Output: sum, carry

Half adder
Input: 2 bits
Output: sum, carry

Q: How do computers perform op's?

Operations

NOT

a	NOT a
0	1
1	0

XOR

a	b	a XOR b
0	0	0
0	1	1
1	0	1
1	1	0

AND

a	b	a AND b
0	0	0
0	1	0
1	0	0
1	1	1

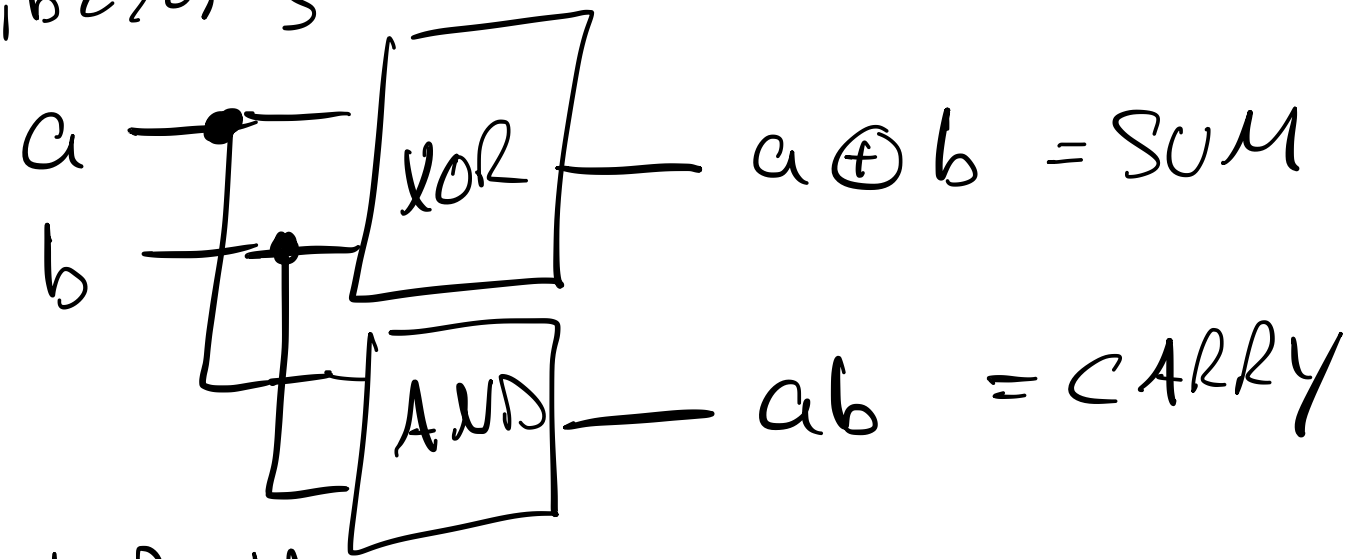
SUM

XOR!

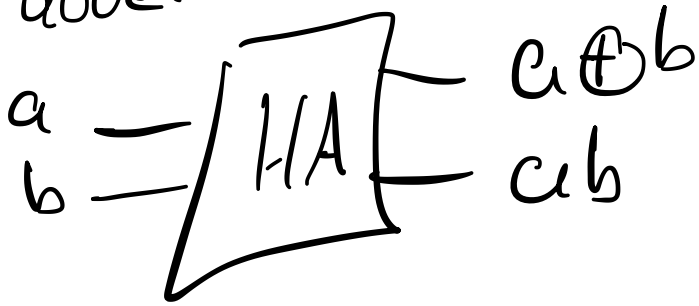
CARRY

AND!

$a, b \in \{0, 1\}$



Half adder



Full adder

a	b	C_{in}	SUM	CARRY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

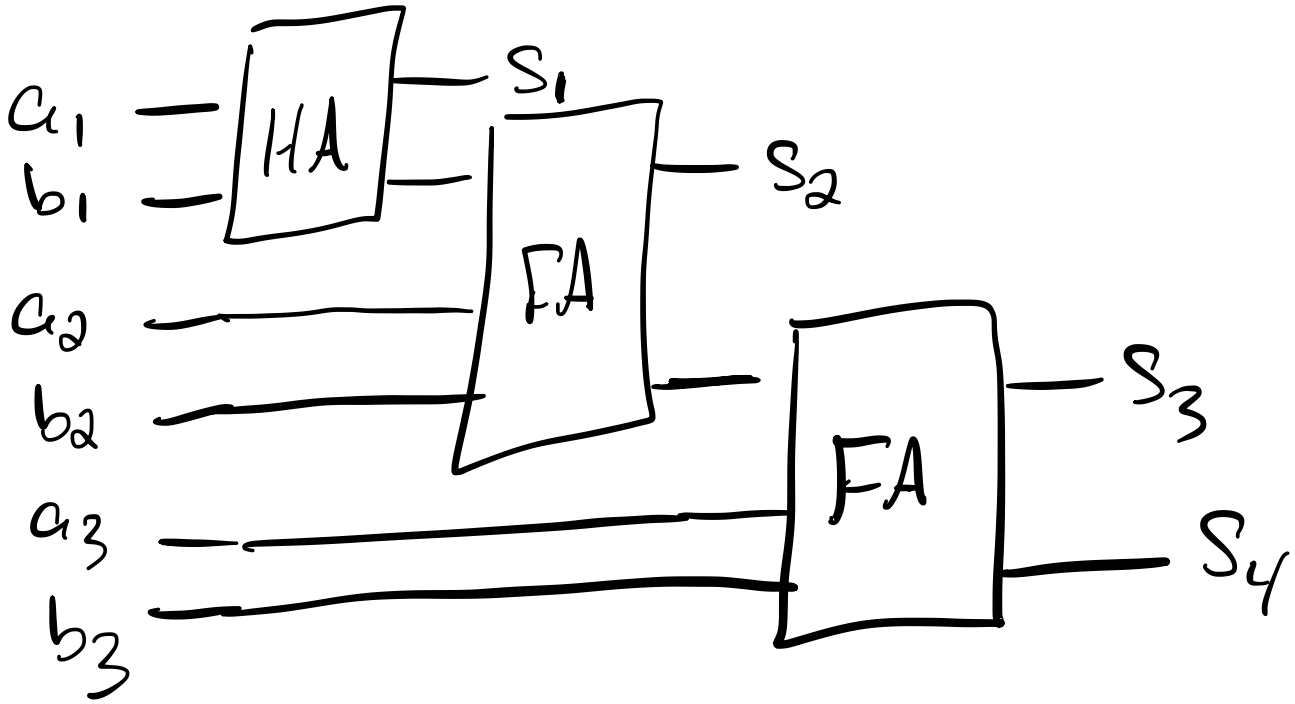
$SUM = a \oplus b \oplus C_{in}$

$CARRY = ab \oplus C_{in}(a \oplus b)$

Ripple Carry adder

$$a = a_3 a_2 a_1 \qquad a_i \in \{0,1\}$$

$$b = b_3 b_2 b_1 \qquad b_i \in \{0,1\}$$



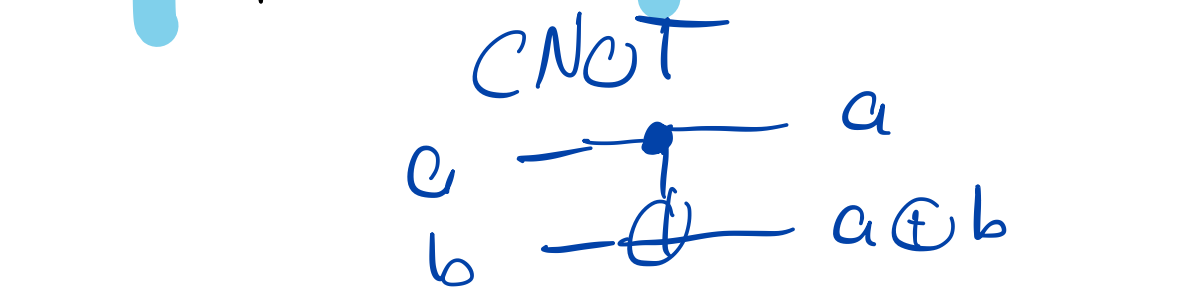
$$S = S_4 S_3 S_2 S_1$$

Quantum

* Logic gates (op's) must be reversible (aka unitary)

Can we quantumly do
- FA?
- HA?
- AND?
- XOR?

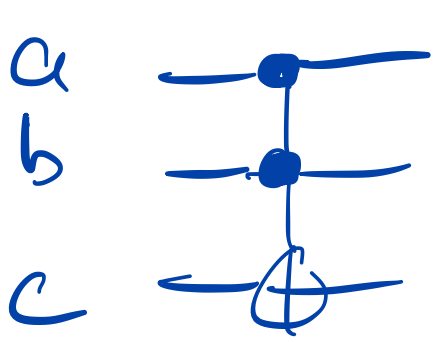
Input		Output	
a	b	a	a ⊕ b
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



AND

a	b	a	b	a AND b
0	0	0	0	0
0	1	0	1	0
1	0	1	0	0
1	1	1	1	1

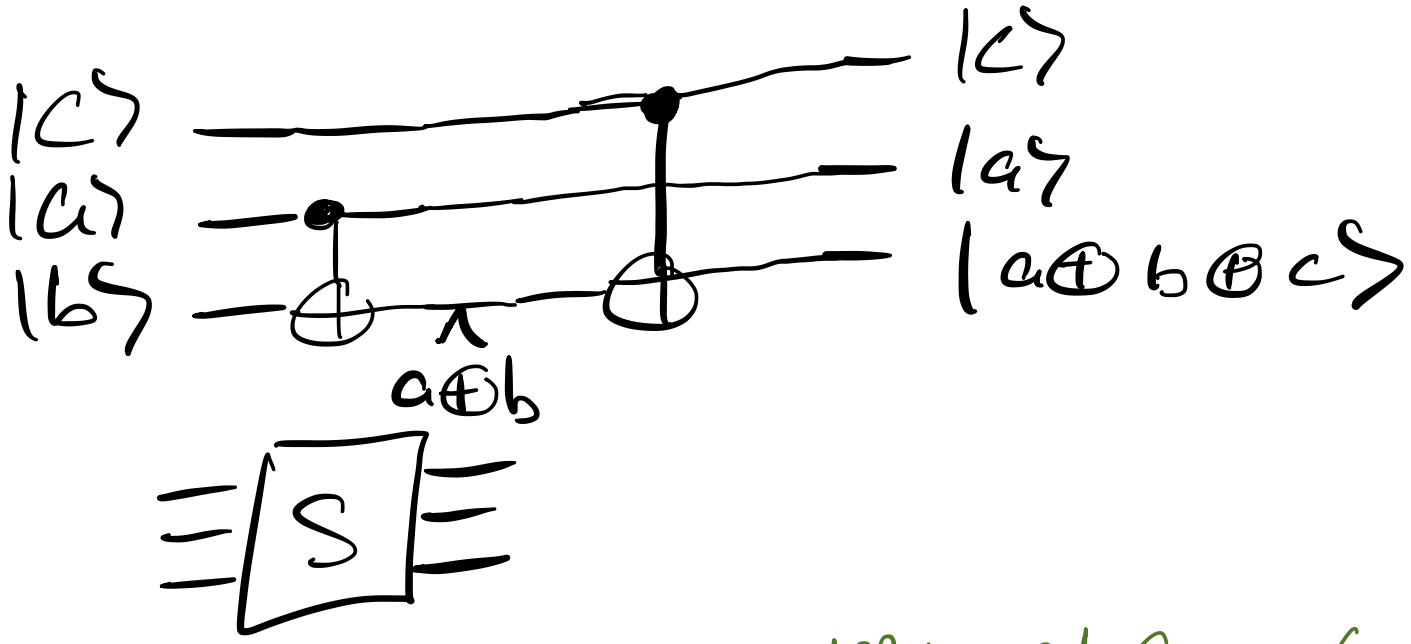
Toffoli

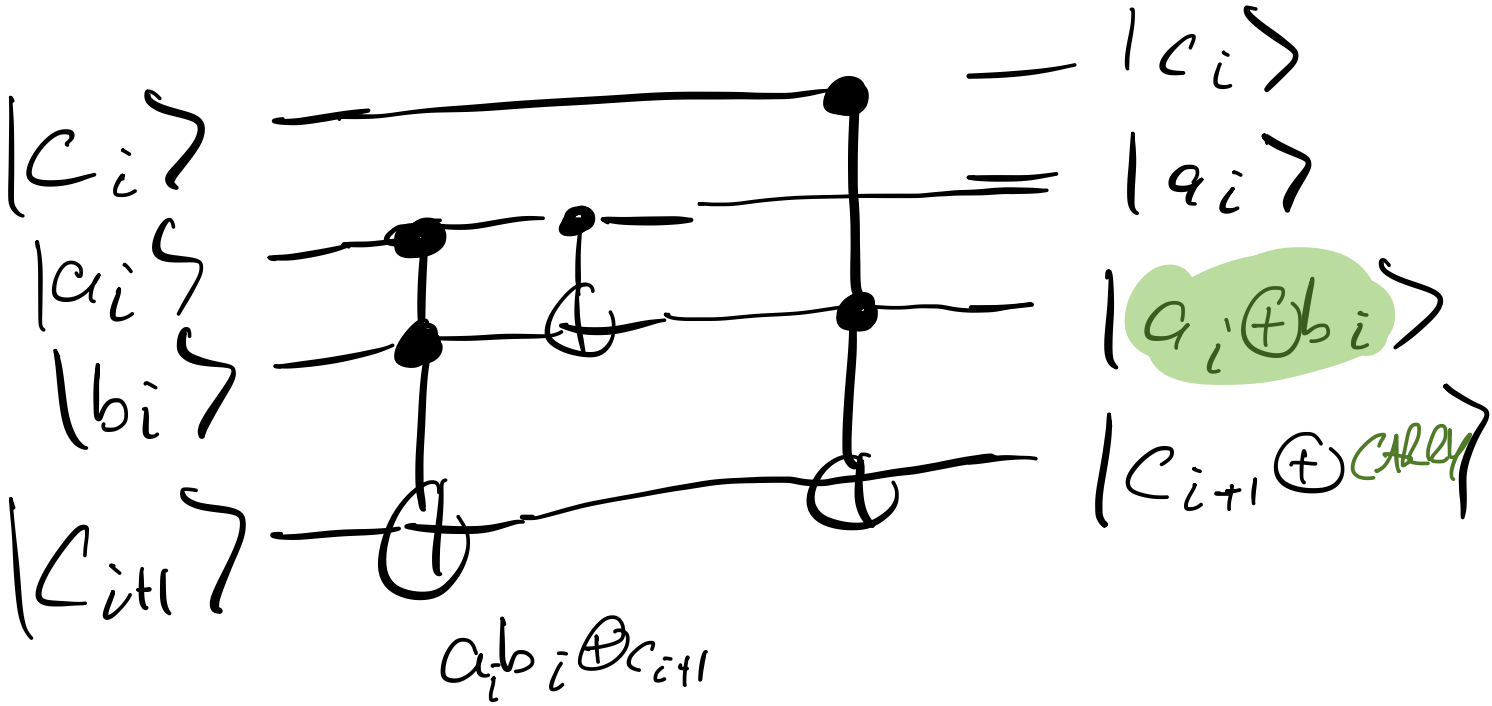


Quantum Sum

CNOT

Toffoli



Quantum Carry

CARRY = ab ⊕ c_in (a⊕b)