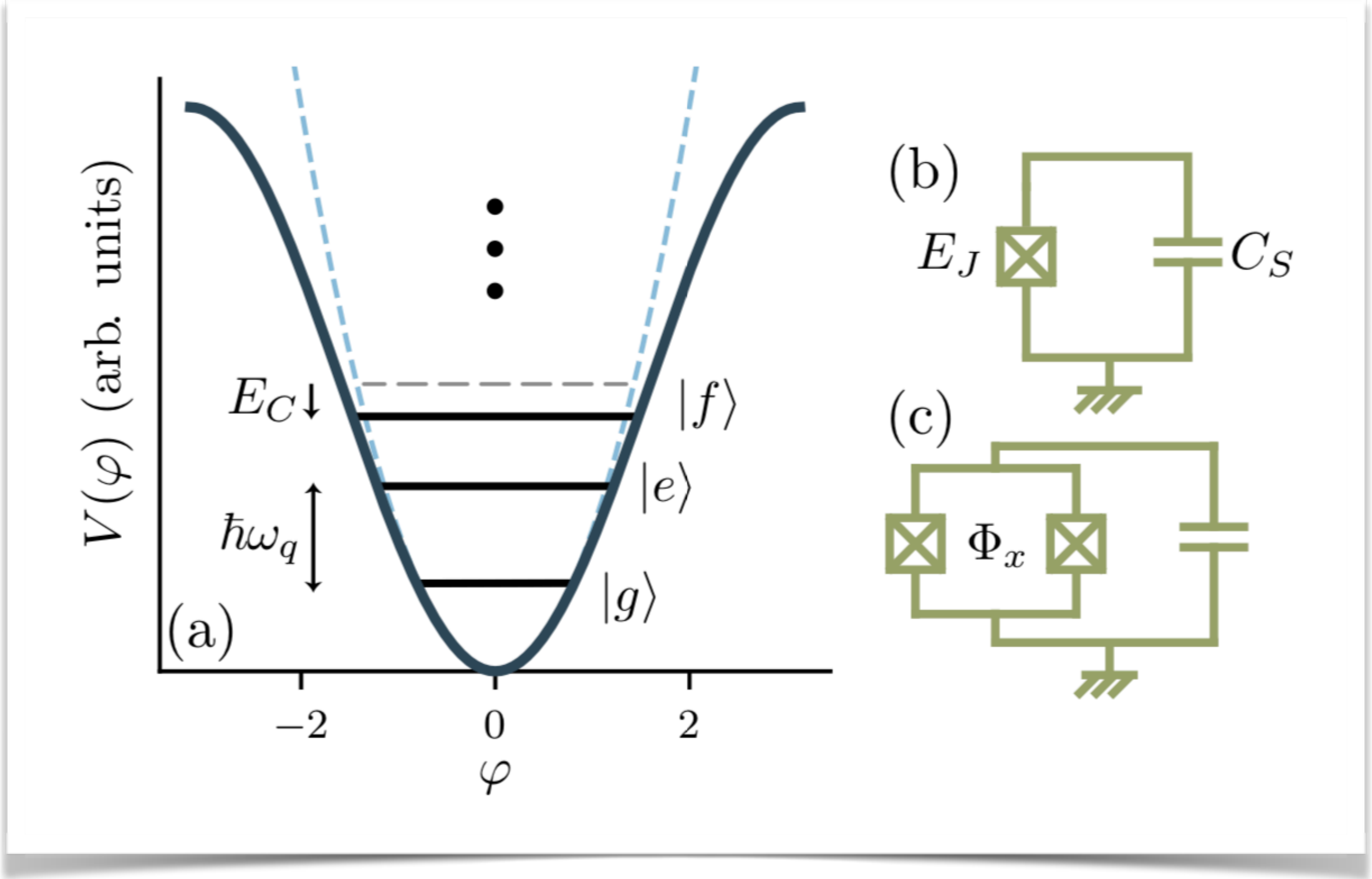
Single and Two-qubit gates in superconducting circuits

A. Blais et al., "Circuit quantum electrodynamics”, *arXiv:2005.12667* (2020)

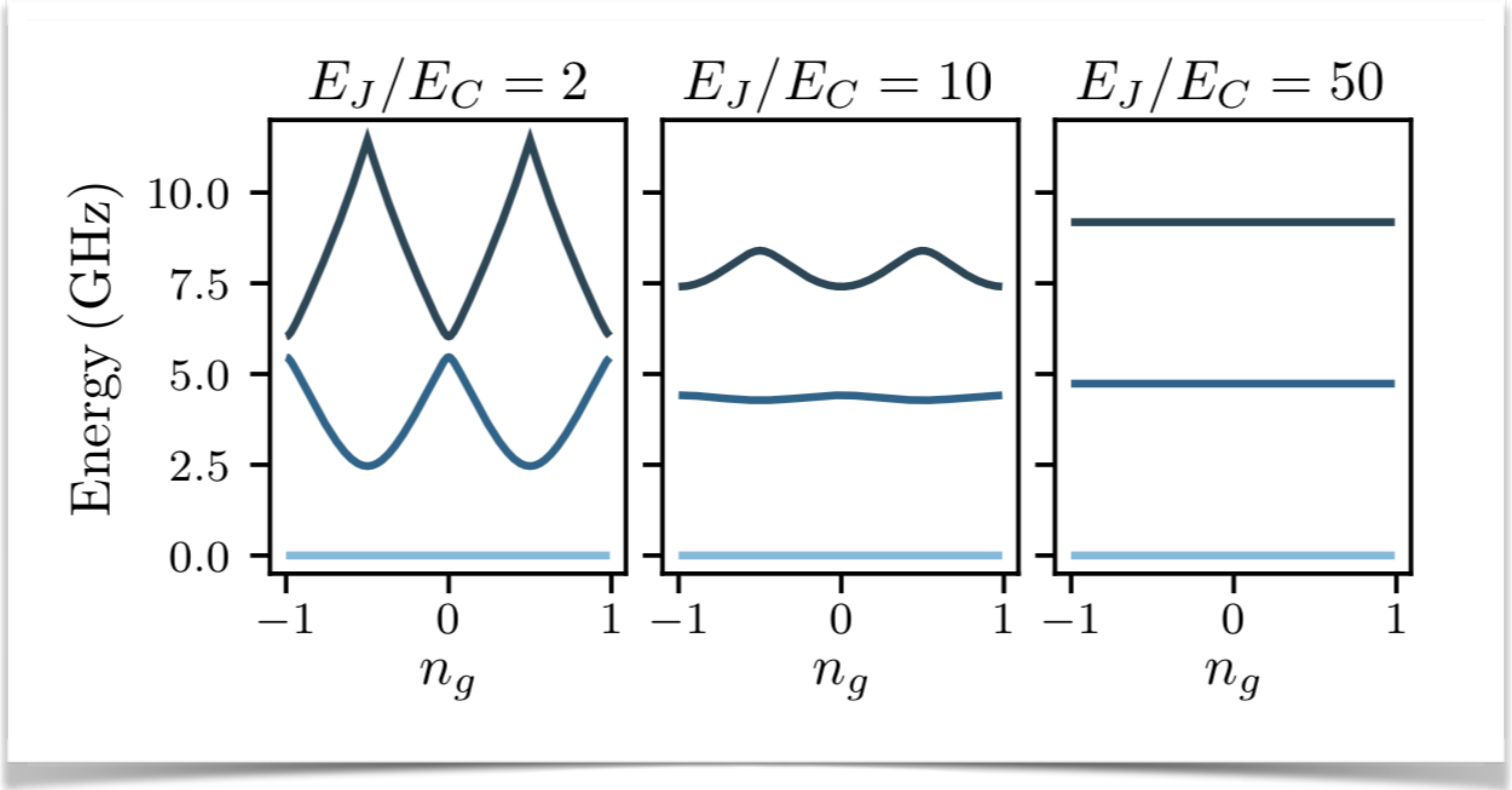
Quantum computing with circuit QED

Transmon qubit

Potential well of the transom qubit



Transmon qubit energy spectrum



$$\hat{H}_{\mathrm{T}} = \frac{(\hat{Q} - Q_g)^2}{2C_\Sigma} - E_J \cos\left(\frac{2\pi}{\Phi_0}\hat{\Phi}\right)$$
$$= 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi}.$$

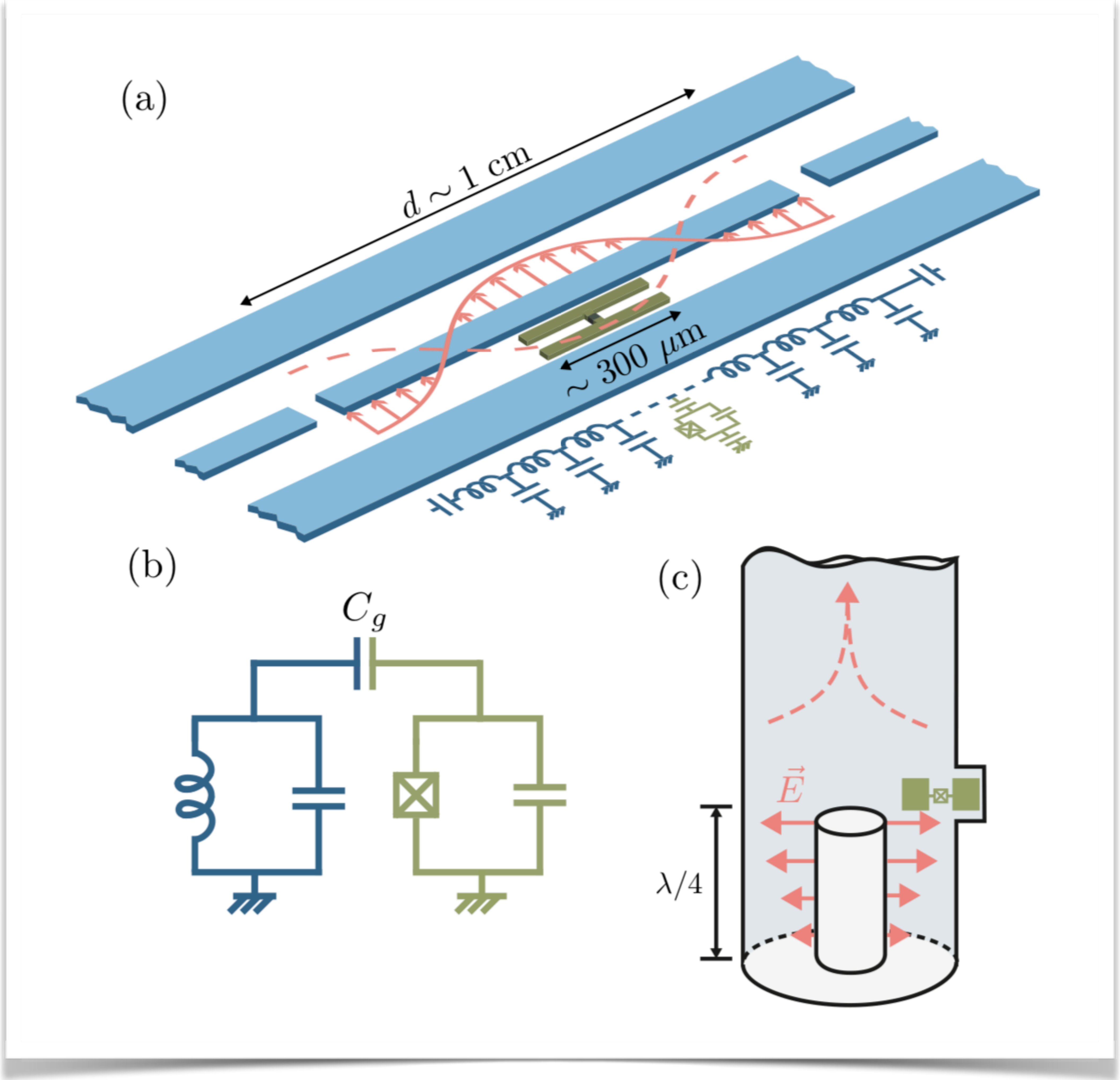
$$\hat{\varphi} = \left(\frac{2E_C}{E_J}\right)^{1/4}(\hat{b}^\dagger + \hat{b}),$$
$$\hat{n} = \frac{i}{2}\left(\frac{E_J}{2E_C}\right)^{1/4}(\hat{b}^\dagger - \hat{b}).$$

$$\hat{H}_{\mathrm{q}} = \sqrt{8E_C E_J}\hat{b}^\dagger\hat{b} - \frac{E_C}{12}(\hat{b}^\dagger + \hat{b})^4$$
$$\approx \hbar\omega_q\hat{b}^\dagger\hat{b} - \frac{E_C}{2}\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b},$$

$$\hbar\omega_q = \sqrt{8E_C E_J} - E_C$$

Quantum computing with circuit QED

Light-matter interaction

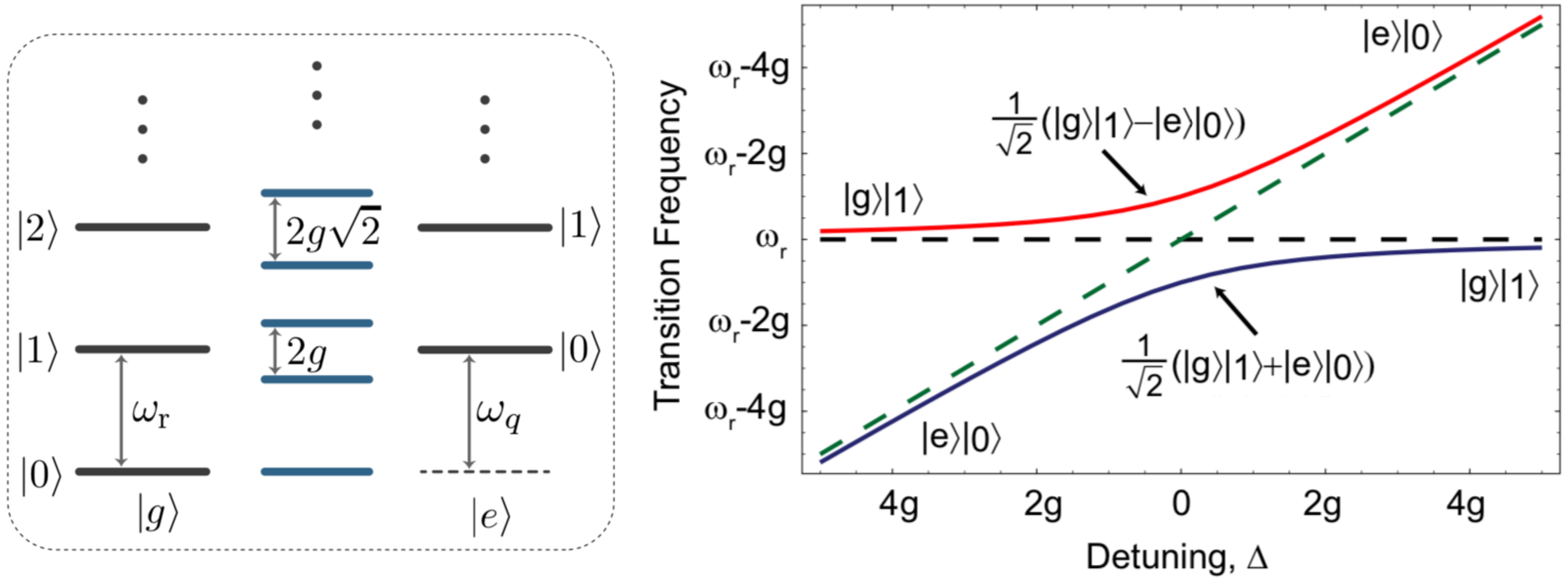


Transmon qubit coupled to a 1D transmission-line resonator

$$\hat{H} \approx \hbar\omega_r \hat{a}^\dagger \hat{a} + \hbar\omega_q \hat{b}^\dagger \hat{b} - \frac{E_C}{2}\hat{b}^\dagger \hat{b}^\dagger \hat{b}\hat{b} + \hbar g(\hat{b}^\dagger \hat{a} + \hat{b}\hat{a}^\dagger).$$

Jaynes-Cumming Hamiltonian:

$$\hat{H}_{\mathrm{JC}} = \hbar\omega_r \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_q}{2}\hat{\sigma}_z + \hbar g(\hat{a}^\dagger \hat{\sigma}_- + \hat{a}\hat{\sigma}_+),$$



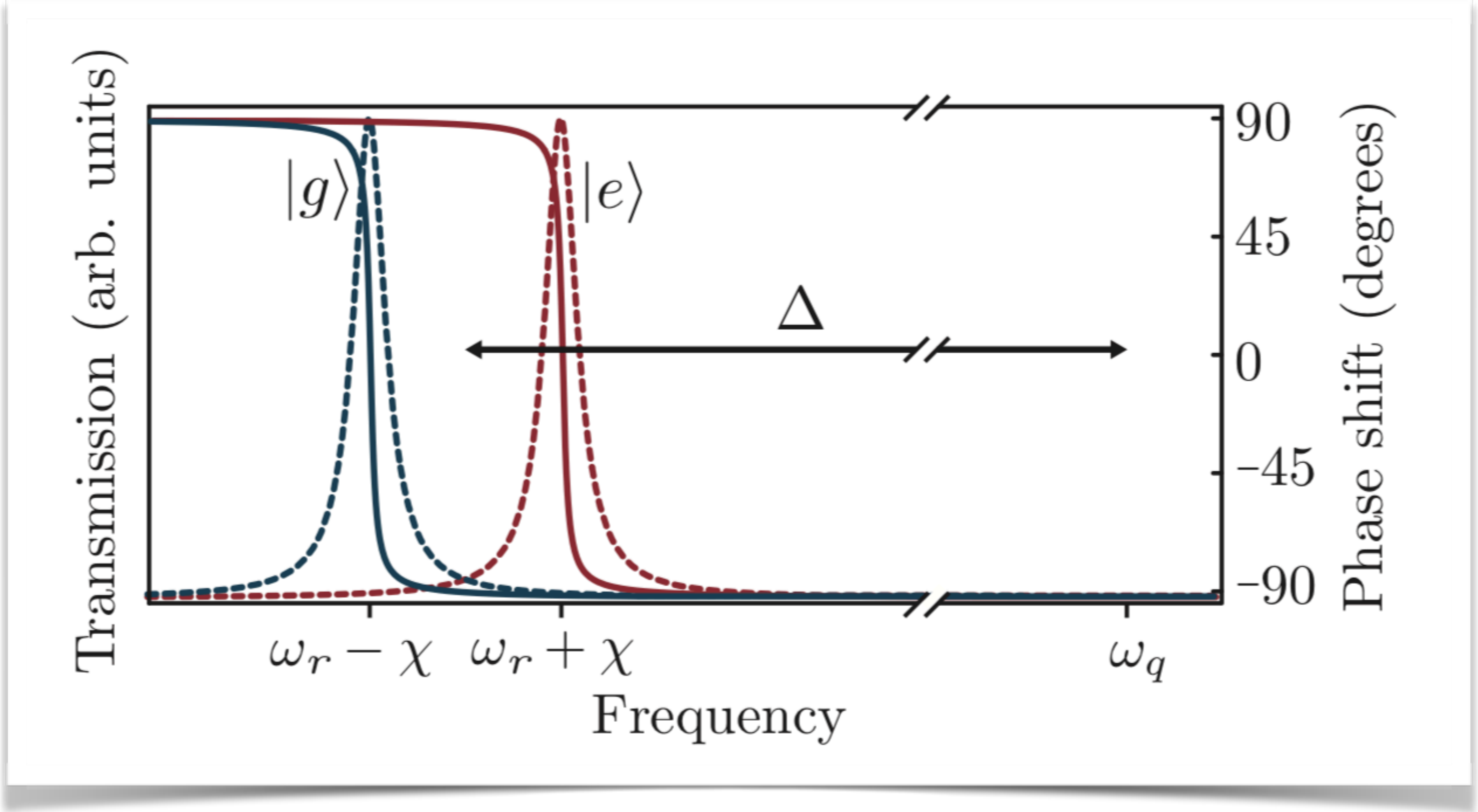
Quantum computing with circuit QED

Dispersive regime

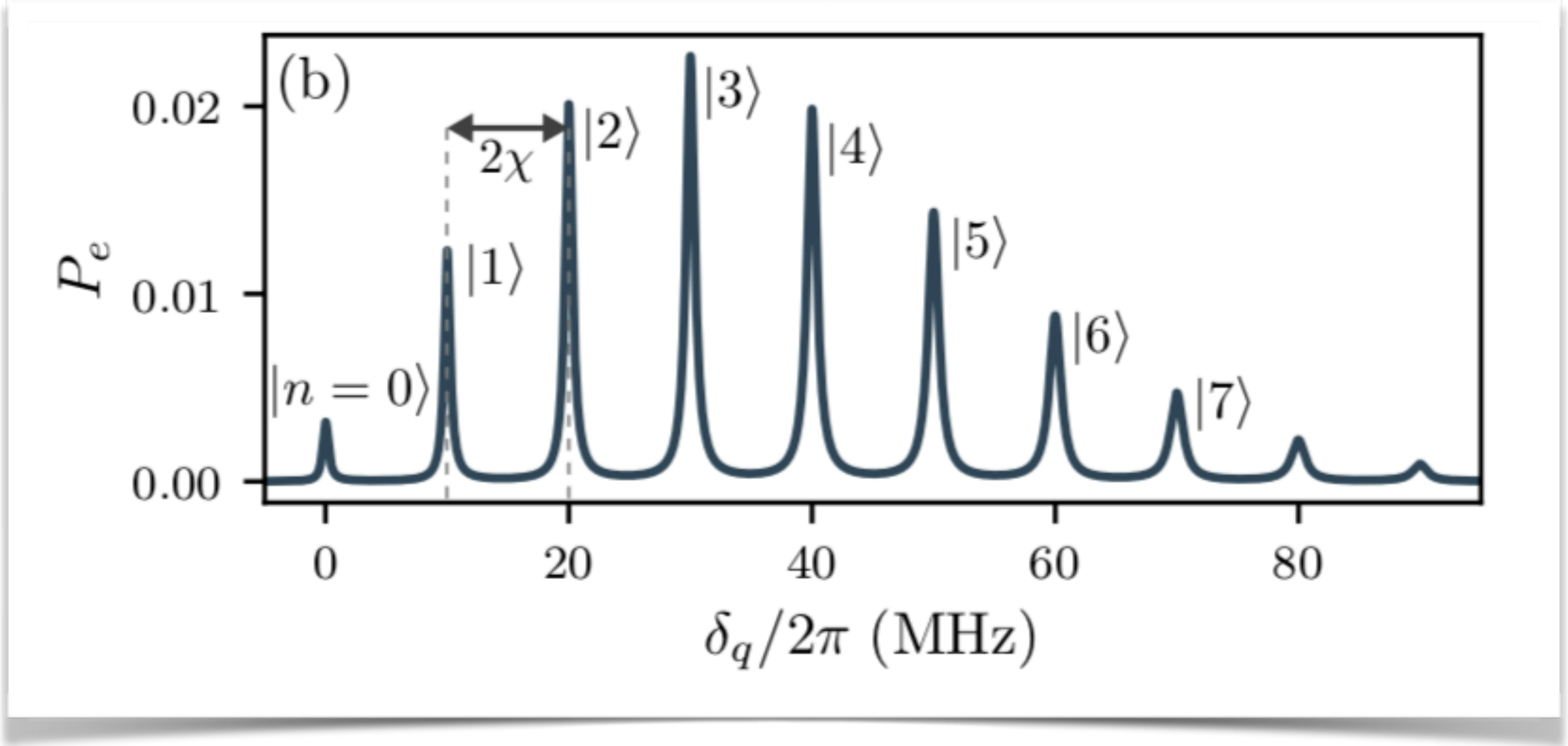
$$|\lambda| = |g/\Delta| \ll 1.$$

$$\hat{H}_{\text{disp}} \approx \hbar\left(\omega_r + \chi\hat{\sigma}_z\right)\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_q}{2}\hat{\sigma}_z. \qquad \chi = -\frac{g^2 E_C/\hbar}{\Delta(\Delta - E_C/\hbar)}.$$

cavity photon number resolved measurement

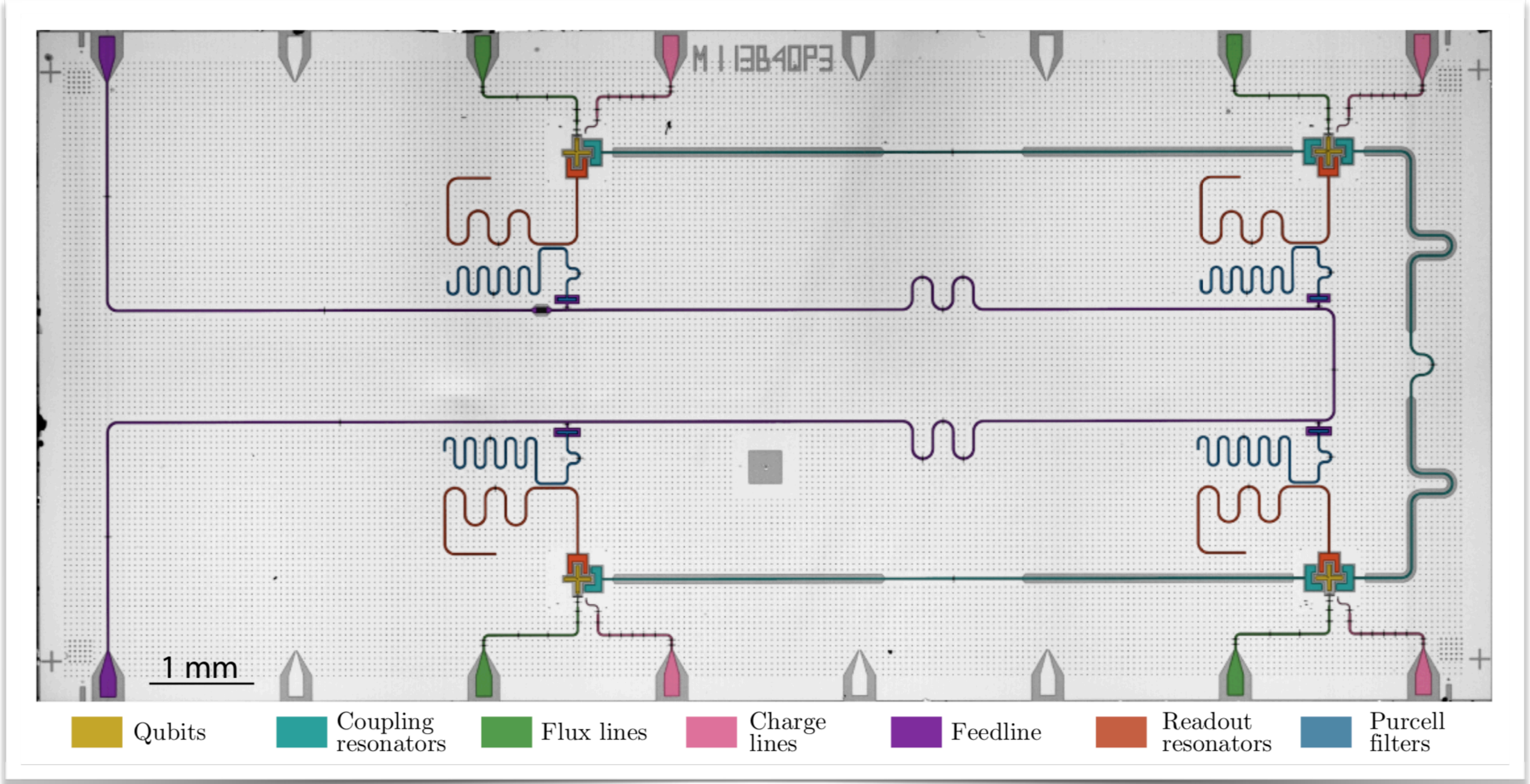


QND measurement



Single qubit control

C.K. Andersen et al., *npj Quantum Information* **5(1),** 1-7 (2019)



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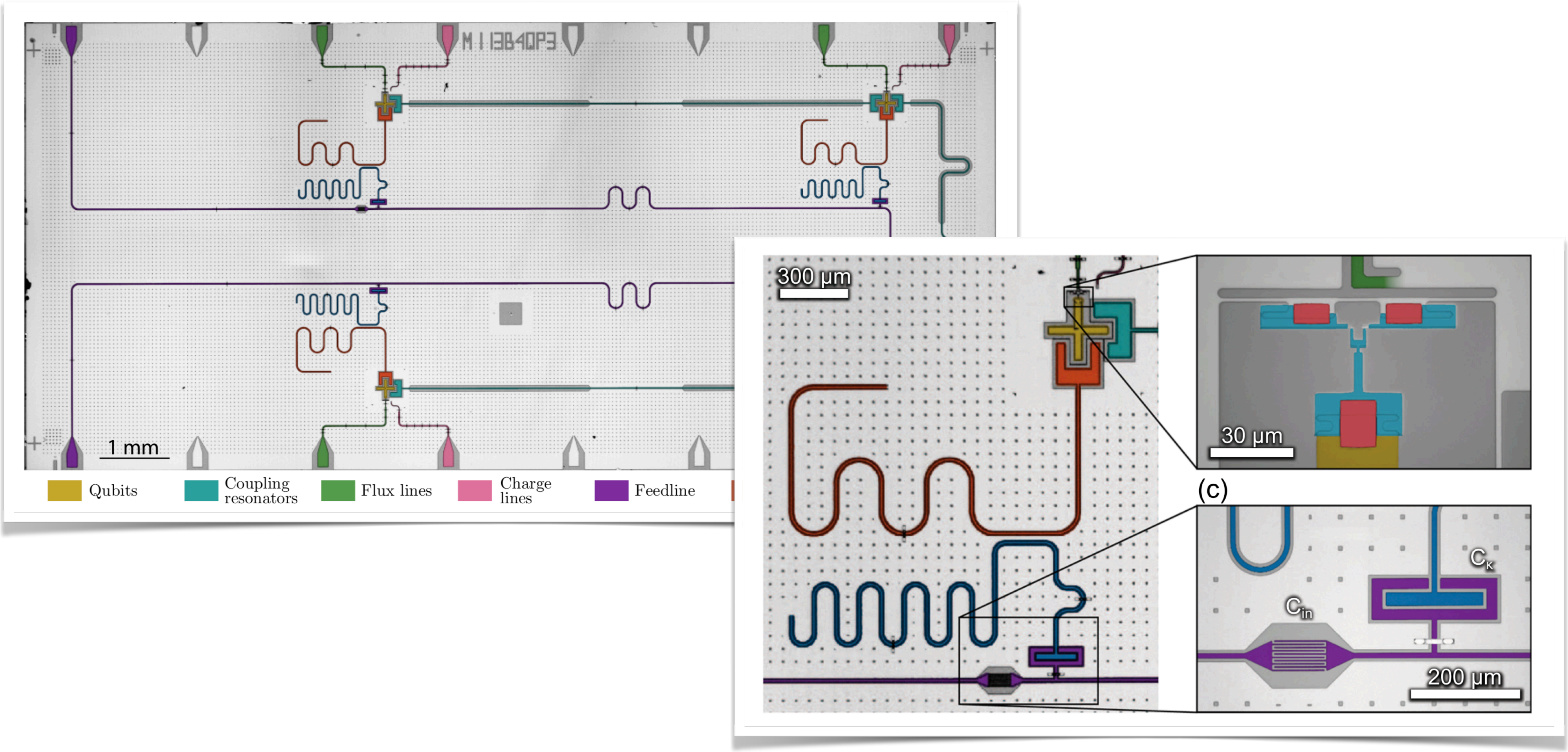

Single qubit control

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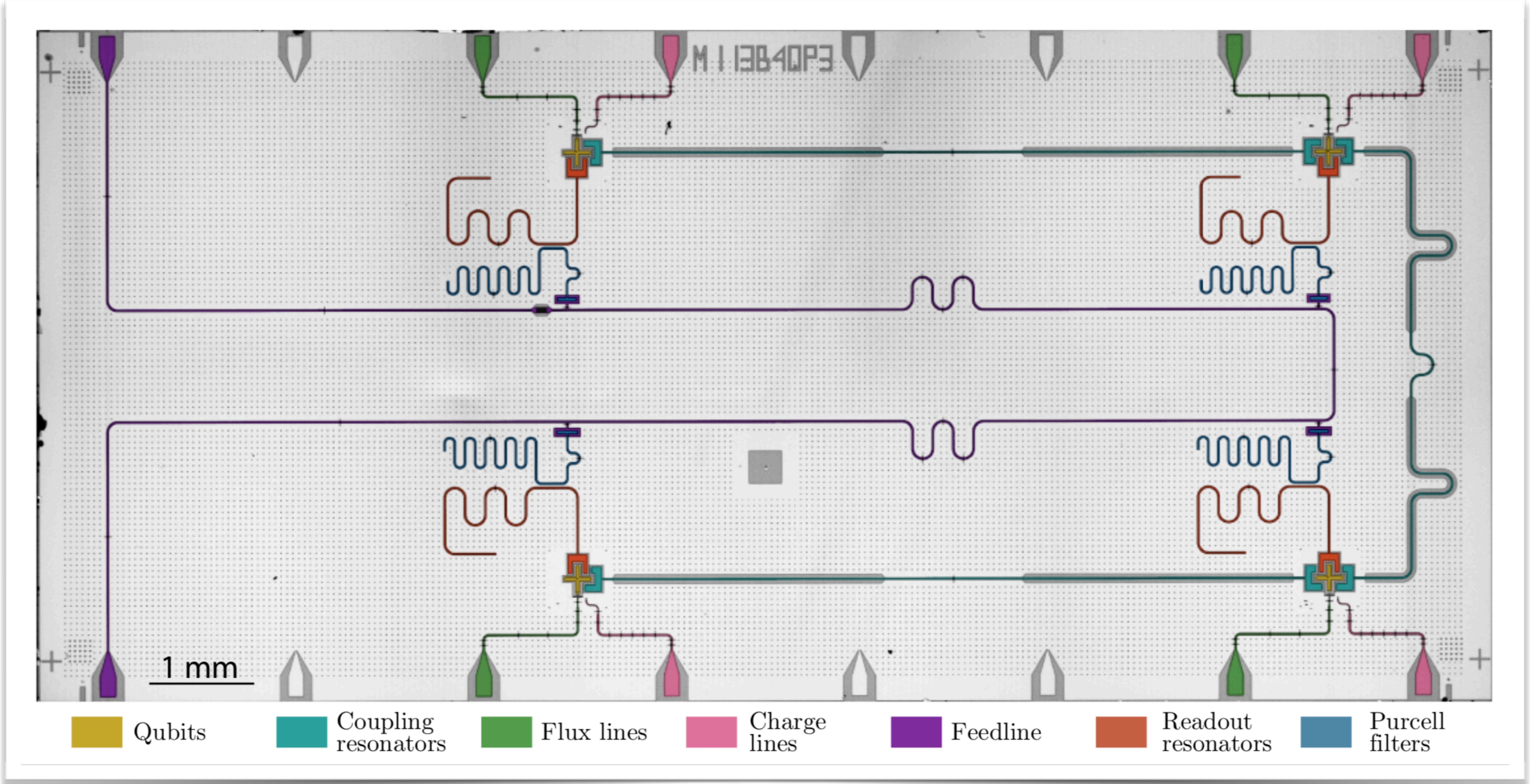
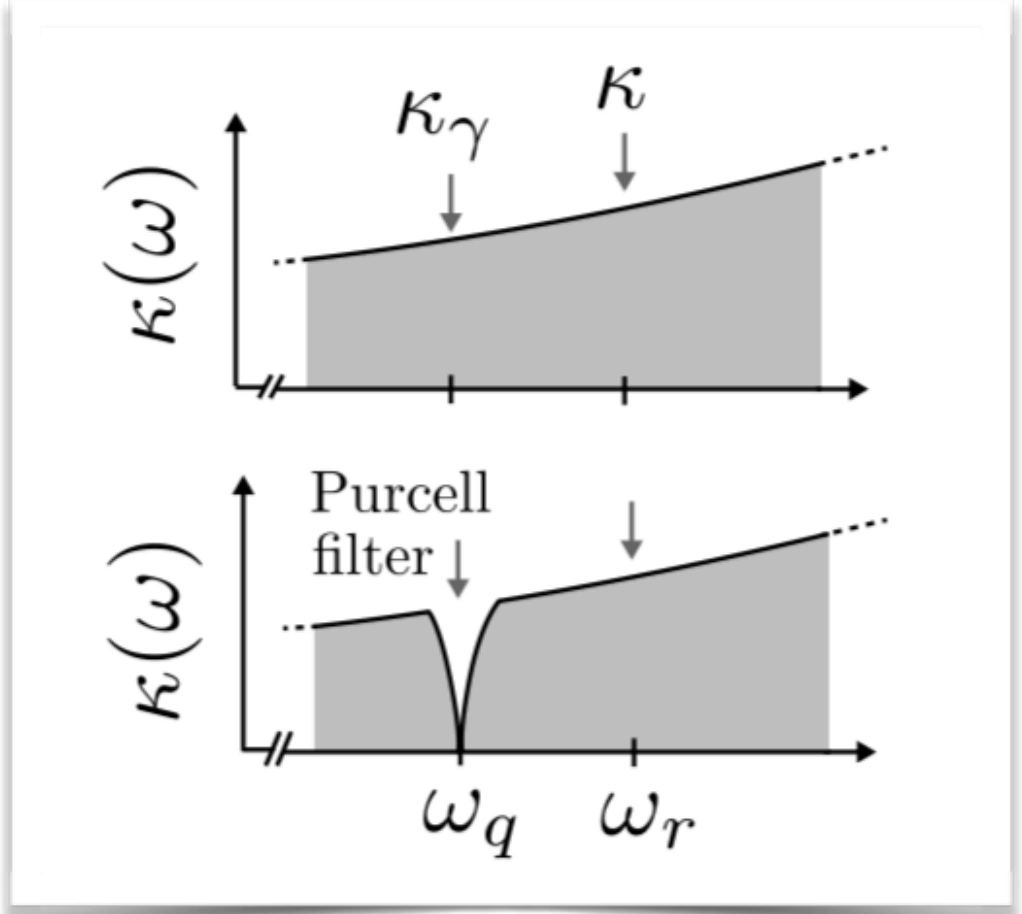




Single qubit control

Qubit Hamiltonian with coherent drive:

$$\hat{H}(t) = \hat{H}_\mathrm{q} + \hbar\varepsilon(t)\left(\hat{b}^\dagger e^{-i\omega_\mathrm{d}t - i\phi_\mathrm{d}} + \hat{b}e^{i\omega_\mathrm{d}t + i\phi_\mathrm{d}}\right)$$

$$\hat{H}_\mathrm{q} = \hbar\omega_q \hat{b}^\dagger\hat{b} - \tfrac{E_C}{2}(\hat{b}^\dagger)^2\hat{b}^2$$

$\downarrow$ *frame rotating at ω_d*

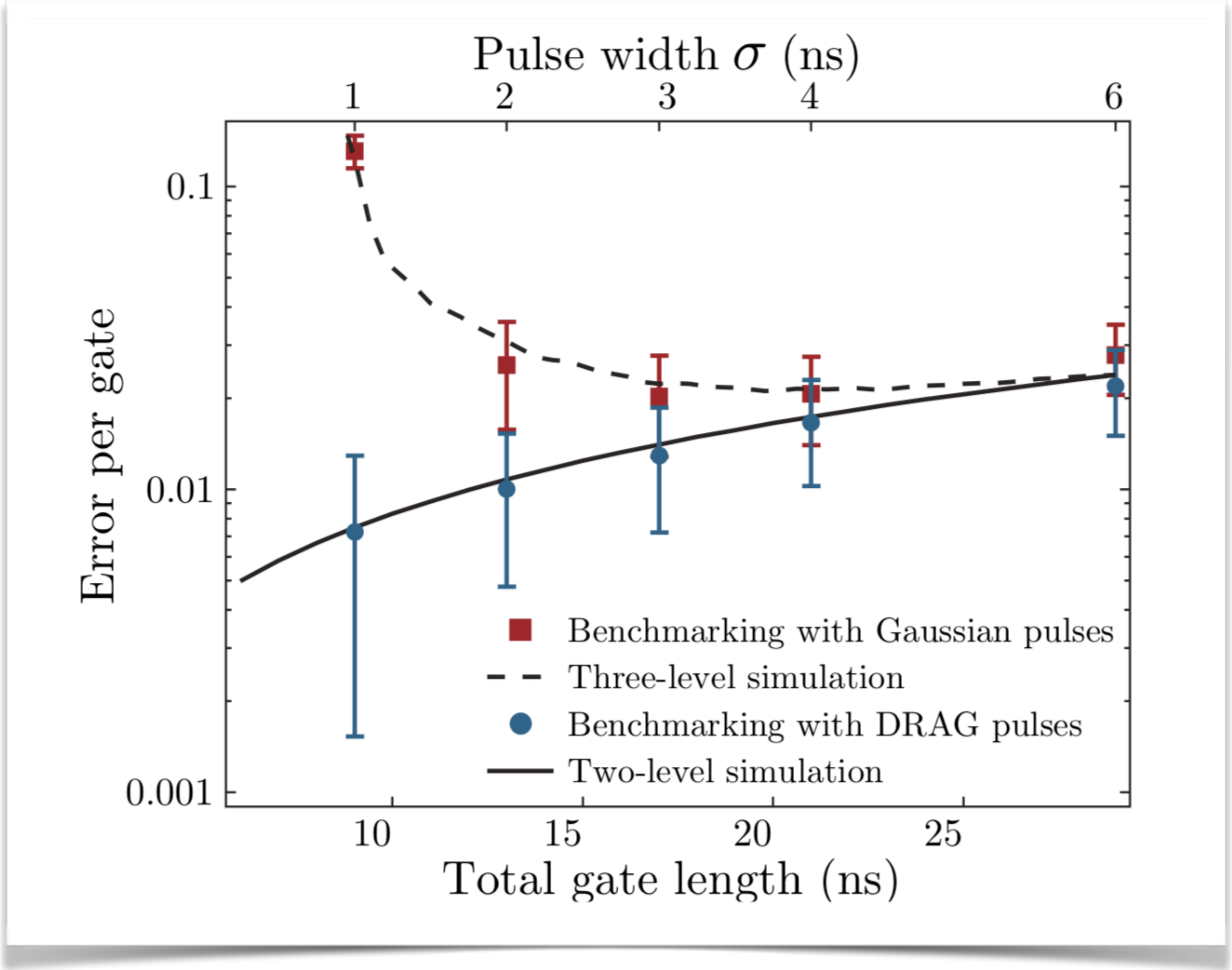
$$\hat{H}' = \hat{H}'_\mathrm{q} + \hbar\varepsilon(t)\left(\hat{b}^\dagger e^{-i\phi_\mathrm{d}} + \hat{b}e^{i\phi_\mathrm{d}}\right)$$

$$\hat{H}'_\mathrm{q} = \hbar\delta_q \hat{b}^\dagger\hat{b} - \tfrac{E_C}{2}(\hat{b}^\dagger)^2\hat{b}^2 \text{ with } \delta_q = \omega_q - \omega_d$$

$\downarrow$ *two level approx.*

$$\hat{H}' = \frac{\hbar\delta_q}{2}\hat{\sigma}_z + \frac{\hbar\Omega_R(t)}{2}\left[\cos(\phi_\mathrm{d})\hat{\sigma}_x + \sin(\phi_\mathrm{d})\hat{\sigma}_y\right]$$

Pulse shaping techniques / DRAG

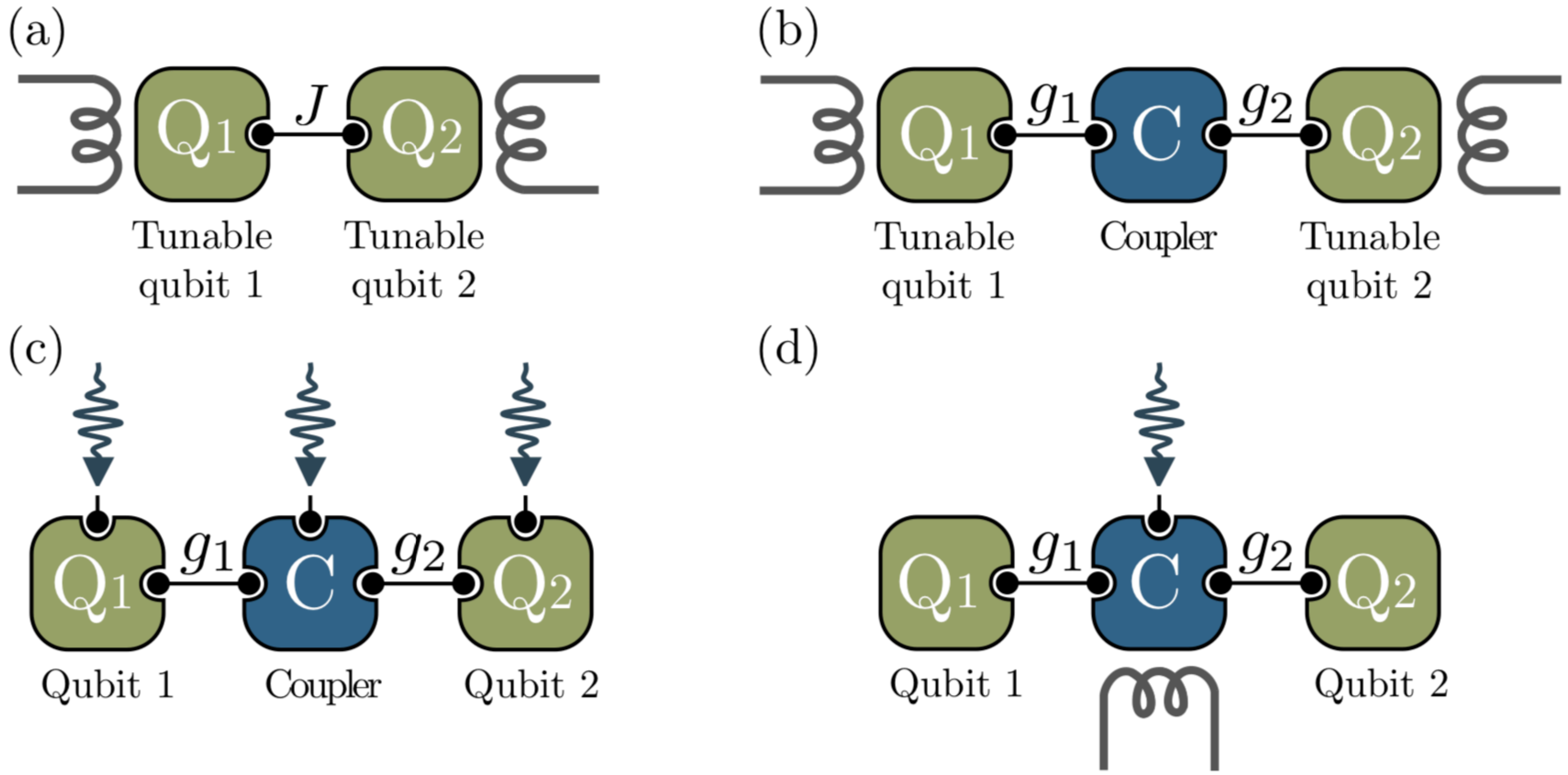


J.M. Chow et al., *Physical Review A* 82.4 (2010): 040305
F. Motzoi et al., *Physical Review Letters* 103.11 (2009): 110501

Two-qubit gates

direct capacitive coupling

exchange interaction mediated by a coupler (bus resonator)



all-microwave gates activated by microwave drives

parametric gates

Qubit-qubit exchange interaction

Direct capacitive coupling

R.C. Bialczak et al., *Nature Physics* **6.6** (2010): 409-413.

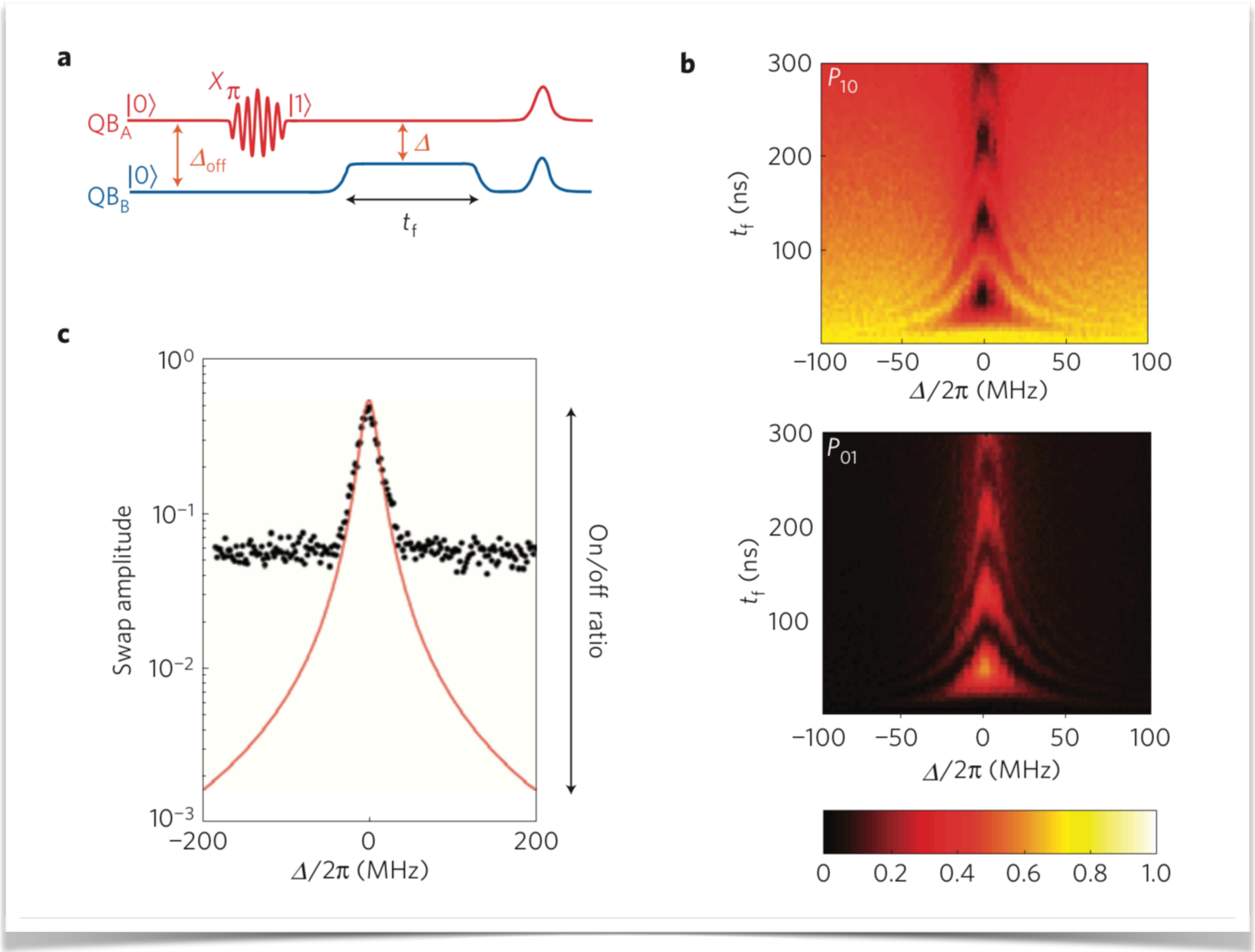
Two-qubit Hamiltonian:

$$\hat{H} = \hat{H}_{q1} + \hat{H}_{q2} + \hbar J(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_1 \hat{b}_2^\dagger),$$

$$\hat{H}_{qi} = \hbar\omega_{qi}\hat{b}_i^\dagger \hat{b}_i - E_{C_i}(\hat{b}_i^\dagger)^2 \hat{b}_i^2/2$$

rotating frame at the qubit frequency:

$$\hat{H}' = \hbar J(\hat{\sigma}_{+1}\hat{\sigma}_{-2} + \hat{\sigma}_{-1}\hat{\sigma}_{+2}).$$



$$U_{\text{int}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(Jt/2) & -i\sin(Jt/2) & 0 \\ 0 & -i\sin(Jt/2) & \cos(Jt/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iSWAP $Jt = \pi$ | √iSWAP $Jt = \pi/2$

$$|10\rangle \rightarrow -i|01\rangle$$

$$|01\rangle \rightarrow -i|10\rangle$$

unwanted coupling $(J^2/\Delta_{12})\hat{\sigma}_{z1}\hat{\sigma}_{z2}$

Qubit-qubit exchange interaction

Resonator mediated coupling

Two-qubit Hamiltonian:

$$\hat{H} = \hat{H}_{q1} + \hat{H}_{q2} + \hbar\omega_r \hat{a}^\dagger \hat{a} + \sum_{i=1}^{2} \hbar g_i (\hat{a}^\dagger \hat{b}_i + \hat{a}\hat{b}_i^\dagger).$$

the effective dispersive Hamiltonian

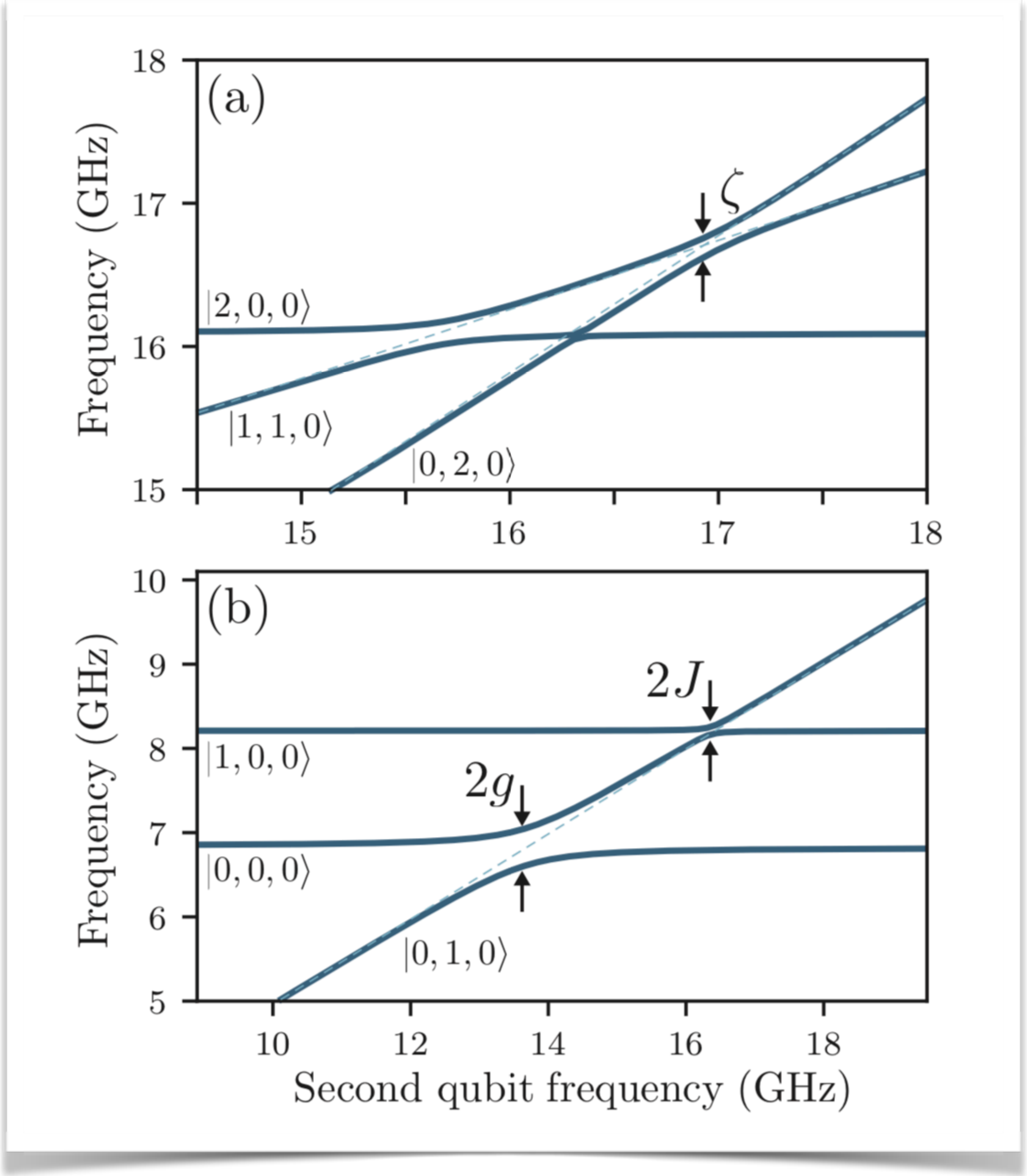
$$H/\hbar = \left(\omega_C + \chi_1\sigma_z^{(1)} + \chi_2\sigma_z^{(2)}\right) a^\dagger a + \frac{1}{2}\omega_1\sigma_z^{(1)} + \frac{1}{2}\omega_2\sigma_z^{(2)} + \frac{g_1 g_2 (\Delta_1 + \Delta_2)}{2\Delta_1\Delta_2}\left(\sigma_+^{(1)}\sigma_-^{(2)} + \sigma_-^{(1)}\sigma_+^{(2)}\right),$$

coupling strength $\quad J = \dfrac{g_1 g_2 (\Delta_1 + \Delta_2)}{2\Delta_1\Delta_2}.$

unwanted ZZ coupling $\quad \zeta \hat{\sigma}_{z1}\hat{\sigma}_{z2}$

$$\zeta = \frac{g_1^2 g_2^2 (\Delta_1 + \Delta_2)}{\Delta_1^2 \Delta_2^2}.$$

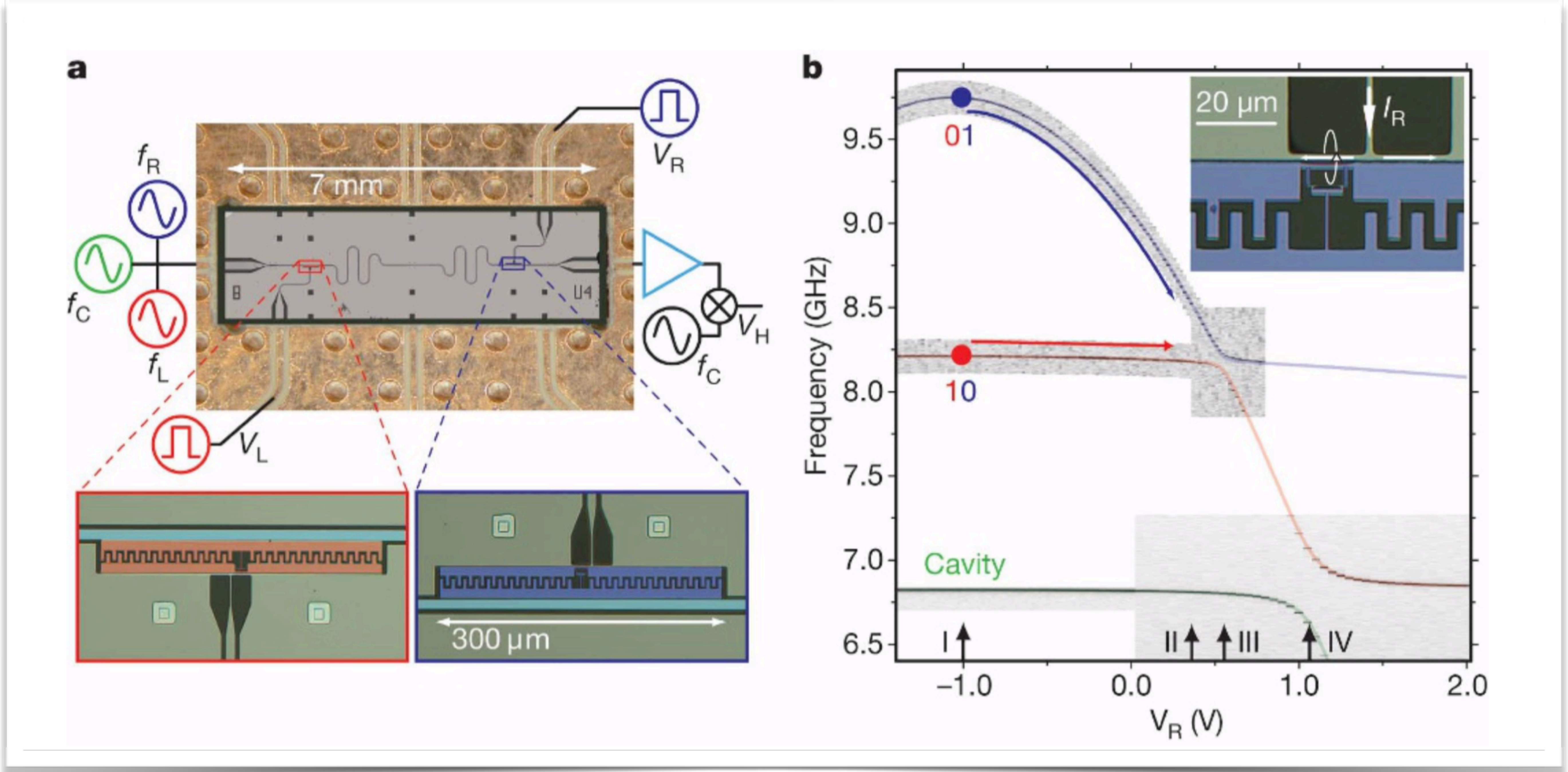
Spectrum of two transom qubits coupled to a common resonator

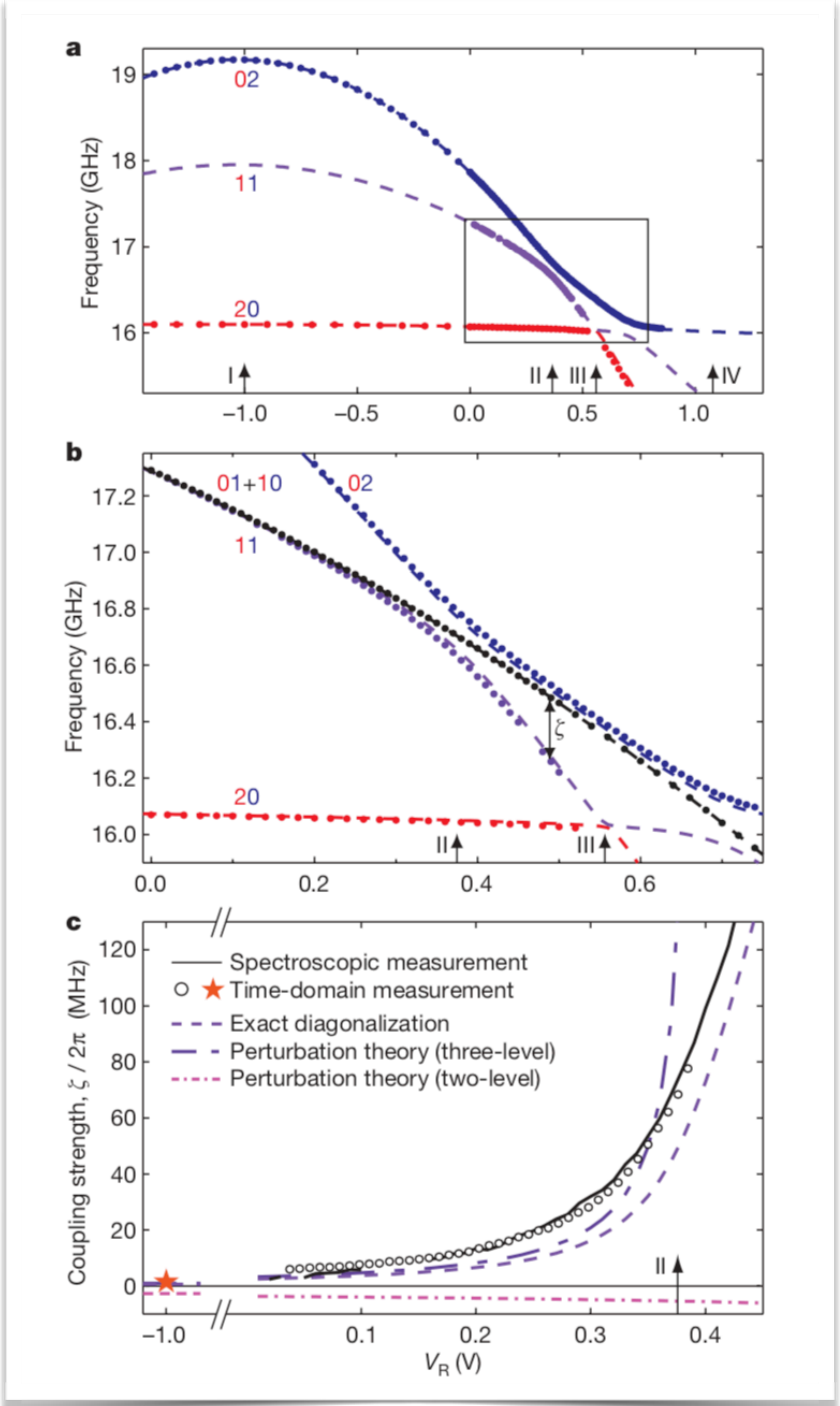


Qubit-qubit exchange interaction

Flux-tuned 11-02 phase gate

L. DiCarlo et al., *Nature* 460.7252 (2009): 240-244.





$$\hat{C}_Z(\phi_{01},\phi_{10},\phi_{11}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\phi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\phi_{11}} \end{pmatrix} \qquad \phi_{ab} = \int dt\, E_{ab}(t)/\hbar$$