Introduction to bosonic codes: binomial codes

M.H. Michael et al. ,Physical Review X 6.3 (2016): 031006

Ling Hu et al., Nature Physics 15.5 (2019): 503-508

Niyaz Beysengulov

Quantum Error Correction (QEC)

Recap

Classical error correction

repetition code

codewords

$$0_L \equiv 000$$
$$1_L \equiv 111$$

physical bit flip probabilities

$$P_0 = (1-\epsilon)^3$$
$$P_1 = 3\epsilon(1-\epsilon)^2$$
$$P_2 = 3\epsilon^2(1-\epsilon)$$
$$P_3 = \epsilon^3$$

majority vote to correct error

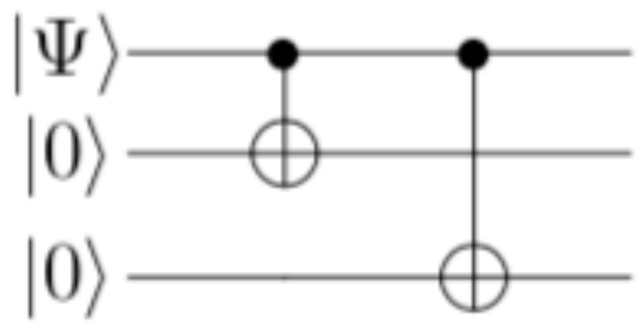
$$110 \longrightarrow 111$$

Quantum error correction

simple example

codewords

$$0_L = |000\rangle$$
$$1_L = |111\rangle$$



Logical Pauli operators

$$X_{log} = X_1 X_2 X_3$$
$$Z_{log} = Z_1 Z_2 Z_3$$
$$Y_{log} = i X_{log} Z_{log}$$

Stabilizers

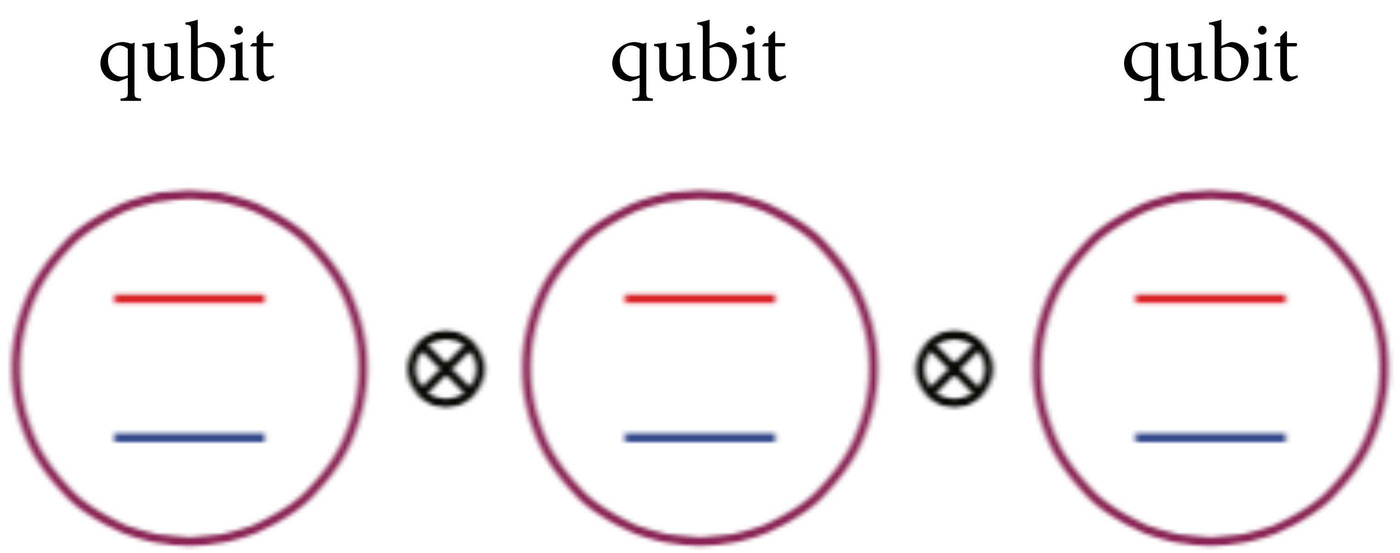
$$S_1 = Z_1 Z_2$$
$$S_2 = Z_2 Z_3$$

Error Type	$\langle S_1 \rangle$	$\langle S_2 \rangle$
I (none)	1	1
X_1	-1	1
X_2	-1	-1
X_3	1	-1

Bosonic mode architecture

Discrete variable

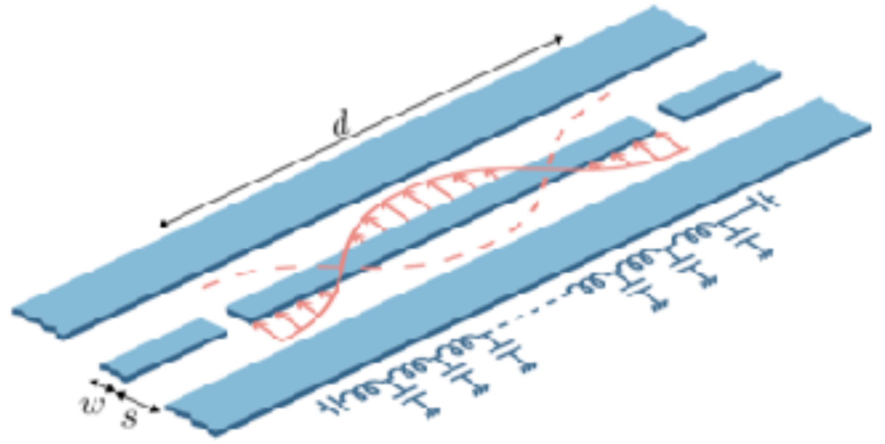
transmon qubits

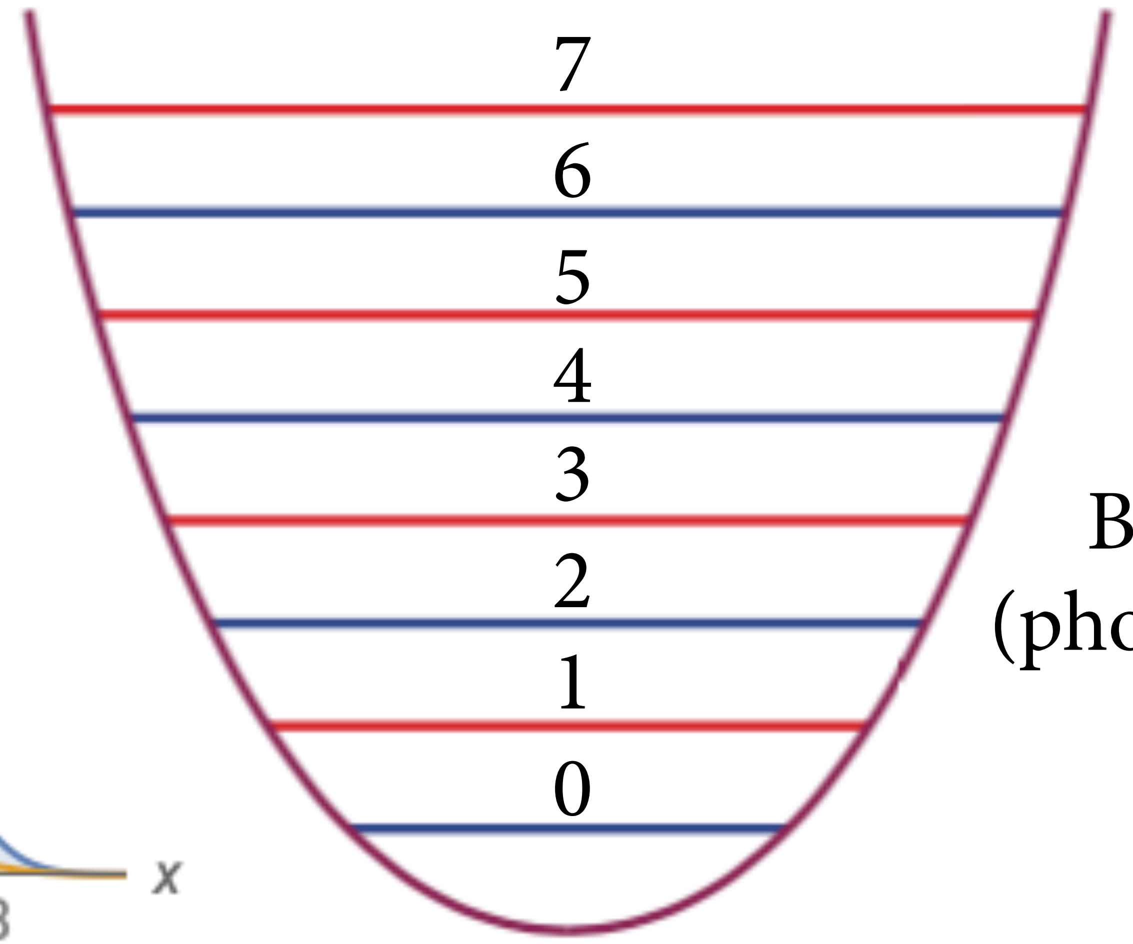


$$|\psi\rangle = a_0|000\rangle + a_1|001\rangle + a_2|010\rangle + a_3|011\rangle$$
$$+a_4|100\rangle + a_5|101\rangle + a_6|110\rangle + a_7|111\rangle$$

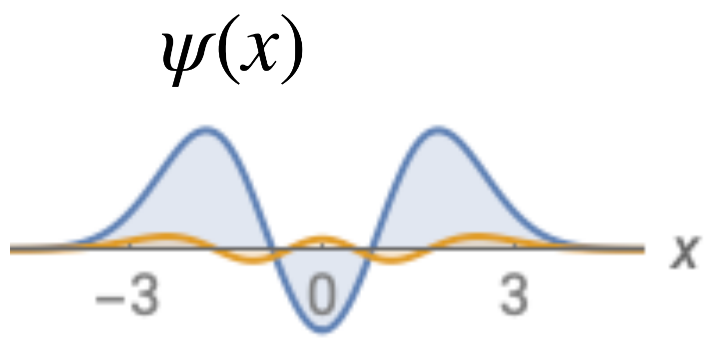
Continuous variable

microwave or mechanical oscillator





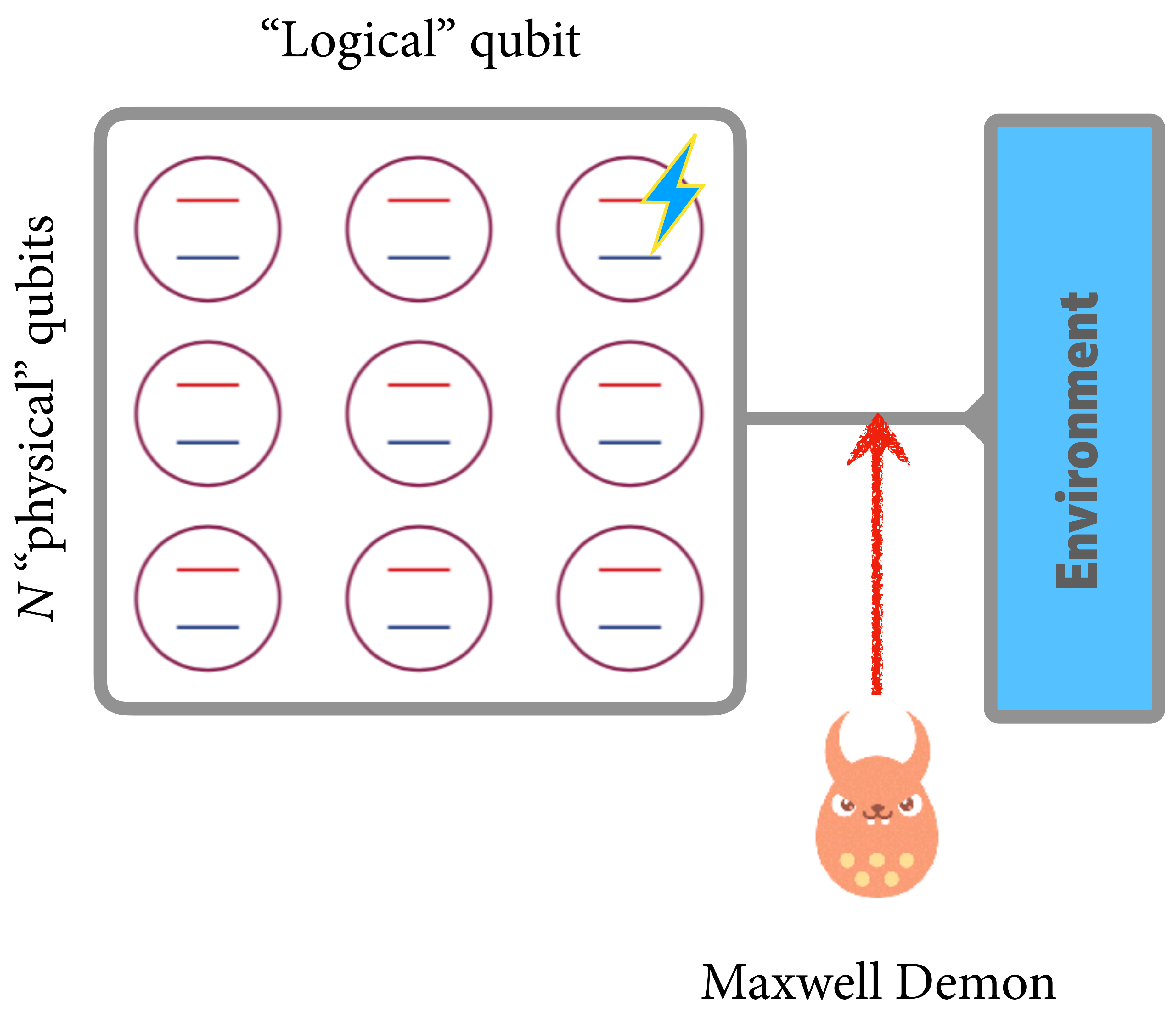
Boson Fock
(photon number)
states



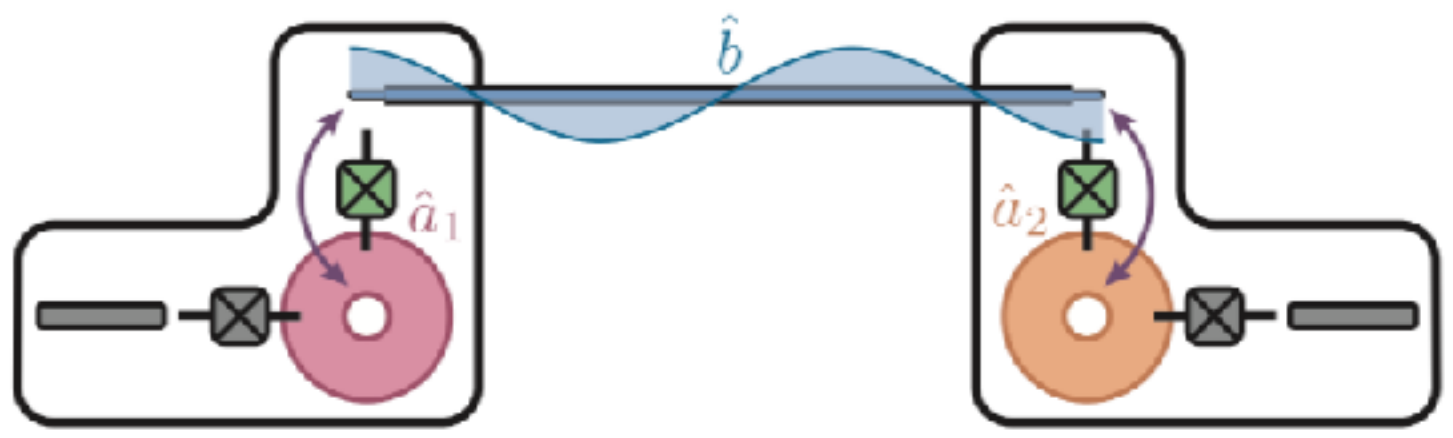
$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle + a_2|2\rangle + a_3|3\rangle$$
$$+a_4|4\rangle + a_5|5\rangle + a_6|6\rangle + a_7|7\rangle$$

Why bosonic codes?

Descrete-variable QEC is hard!



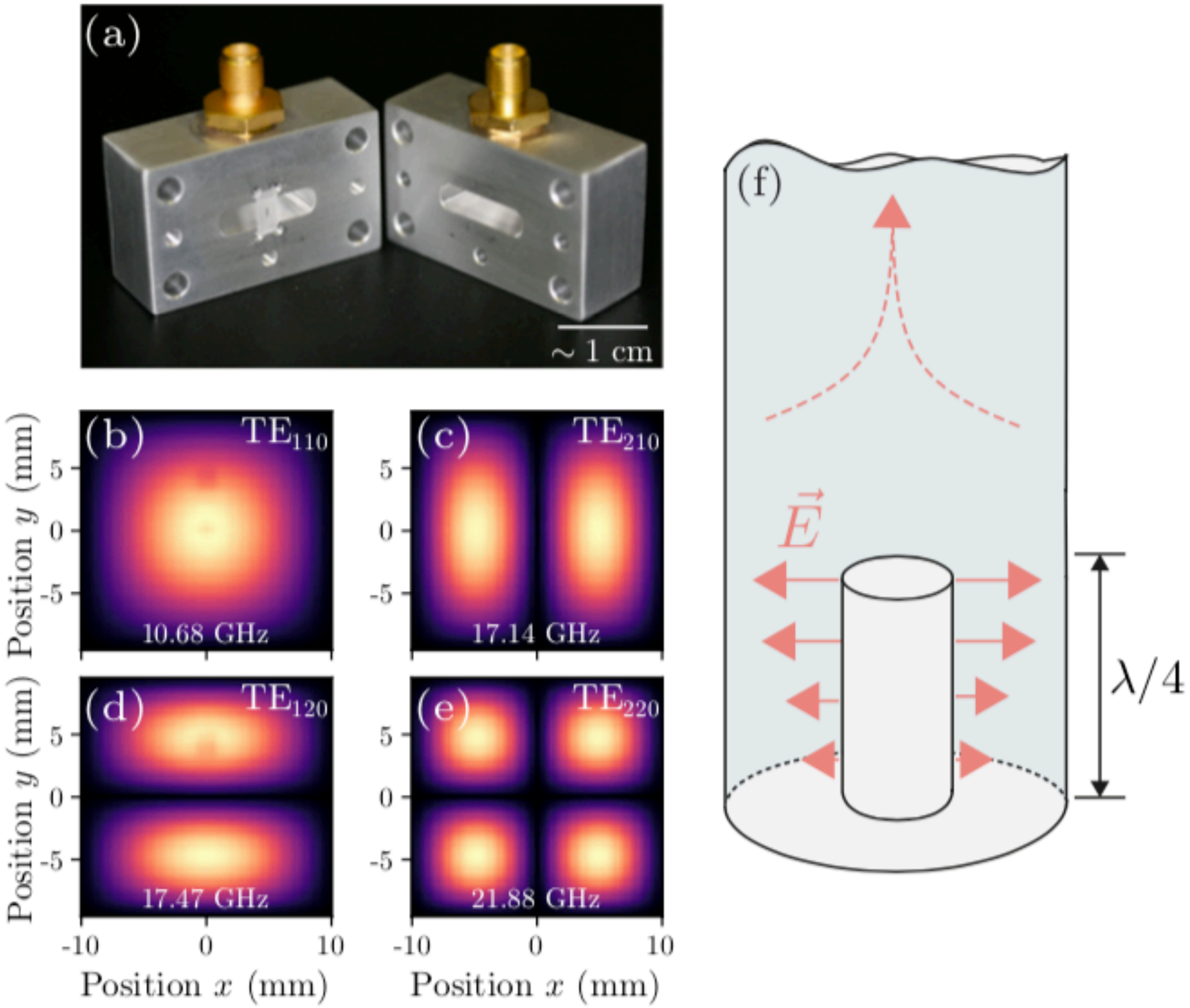
Bosonoc QEC code words of photons in resonators
can be transmitted as 'flying' photons
for QEC local quantum communication



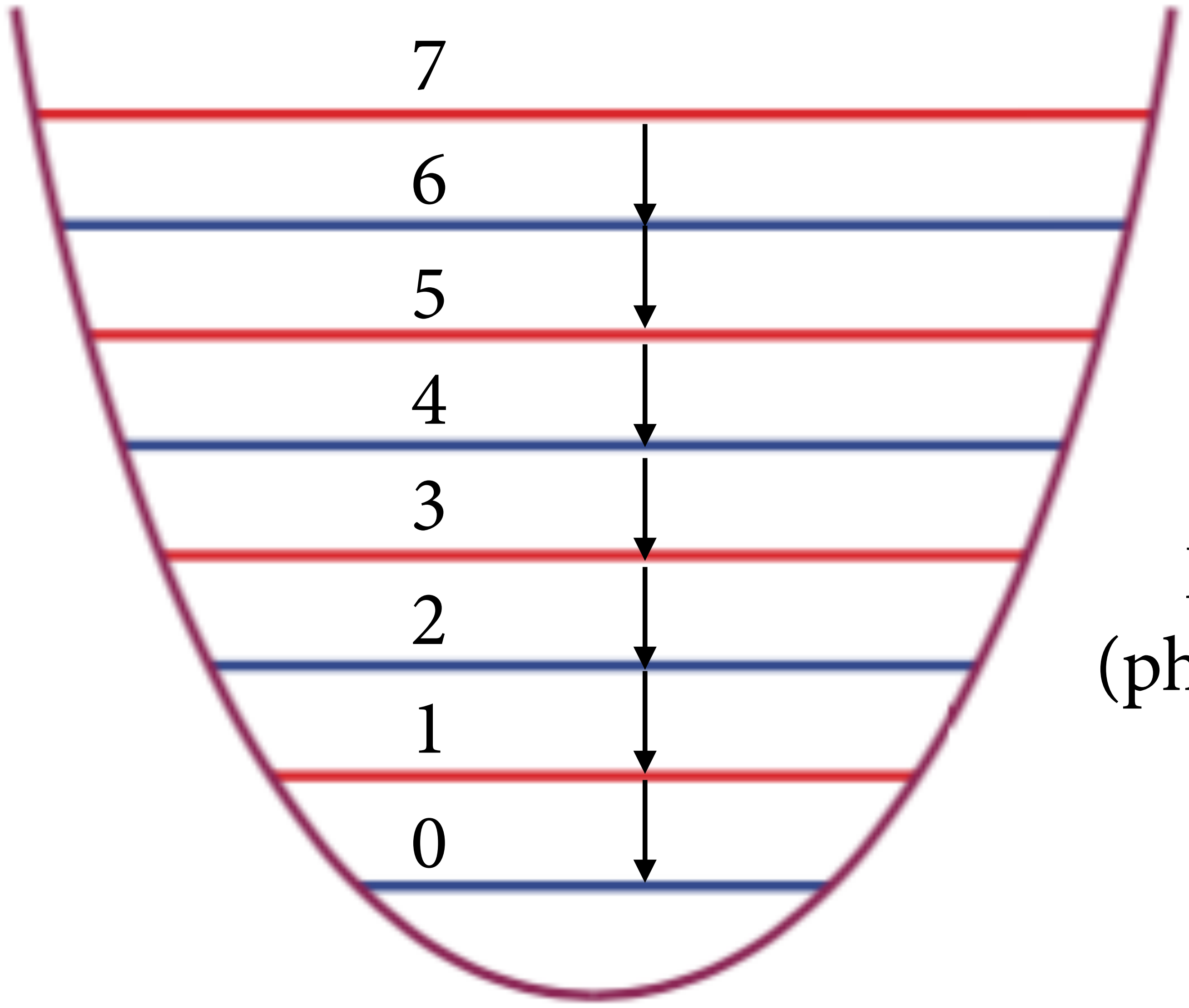
"Error-detected state transfer and entanglement
in a superconducting quantum network"
L.Bukhardt et al., arxiv:2004.06168

Binomial code

microwave resonators (harmonic oscillators)
are empty boxes



$$H = \hbar\omega a^\dagger a = \hbar\omega\hat{n}$$



Boson Fock
(photon number)
states

- simple error model: photon loss
- Codewords with definite photon number parity (e.g. even)
- photon loss flips the parity
- measurement (QND) of the parity does not tell us the photon number

Simple binomial code

Using only 5 photon states 0-4

Logical code words

even parity

$$|0_L\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}}$$

$$|1_L\rangle = |2\rangle$$

Error words

odd parity

$$a|0_L\rangle = \sqrt{2}|3\rangle$$

$$a|1_L\rangle = \sqrt{2}|1\rangle$$

Recovery after parity jump

$$U|3\rangle = |0_L\rangle$$

$$U|1\rangle = |1_L\rangle$$

Correct errors to the first order in κdt

time evolution of the cavity: $$d\hat{\rho} = \kappa dt \left(\hat{a}\,\hat{\rho}\,\hat{a}^\dagger - \frac{\hat{a}^\dagger\hat{a}}{2}\hat{\rho} - \hat{\rho}\frac{\hat{a}^\dagger\hat{a}}{2} \right)$$

exact error operators:
(Kraus operators)

$$\hat{E}_\ell = \sqrt{\frac{(1 - e^{-\kappa\delta t})^\ell}{\ell!}} e^{-(\kappa\delta t/2)\hat{n}}\hat{a}^\ell$$

first order in κdt

"no jump" $\hat{E}_0 = \sqrt{\hat{I} - \kappa dt\hat{n}}$

"jump" $\hat{E}_1 = \sqrt{\kappa dt a}$

no jump evolution:

$$\frac{|0\rangle + |4\rangle}{\sqrt{2}} \rightarrow \cos\Theta\frac{|0\rangle + |4\rangle}{\sqrt{2}} + \sin(\Theta)\frac{|0\rangle - |4\rangle}{\sqrt{2}}$$

$$|2\rangle \rightarrow |2\rangle$$

Simple binomial code

correcting dephasing

protecting against errors:

$$\bar{\mathcal{E}}_2 = \{\hat{I}, \hat{a}, \hat{a}^2, \hat{n}\}$$

codewords:

$$|W_\uparrow\rangle = \frac{|0\rangle + \sqrt{3}|6\rangle}{2}, \quad |W_\downarrow\rangle = \frac{\sqrt{3}|3\rangle + |9\rangle}{2}.$$

error words:

$$|\bar{E}_\uparrow^1\rangle = |5\rangle \quad \text{and} \quad |\bar{E}_\downarrow^1\rangle = (|2\rangle + |8\rangle)/\sqrt{2}$$

initial quantum state:

$$|\psi\rangle = u|W_\uparrow\rangle + v|W_\downarrow\rangle$$

the dephasing error does not change the photon number

$$|\psi_n\rangle = \hat{n}|\psi\rangle / \sqrt{\langle\psi|\hat{n}^2|\psi\rangle},$$

$$|\psi_n\rangle = u\frac{\sqrt{3}|W_\uparrow\rangle - |\bar{E}_\uparrow^n\rangle}{2} + v\frac{\sqrt{3}|W_\downarrow\rangle - |\bar{E}_\downarrow^n\rangle}{2},$$

in order to correct this we perform projective measurement into logical basis:

$$\hat{P}_{\mathrm{W}} = \sum_\sigma |W_\sigma\rangle\langle W_\sigma|,$$

Binomial code

General case

protecting against error set:

$$\bar{\mathcal{E}} = \{\hat{I}, \hat{a}, \hat{a}^2, \ldots, \hat{a}^L, \hat{a}^\dagger, \ldots, (\hat{a}^\dagger)^G, \hat{n}, \hat{n}^2, \ldots, \hat{n}^D\},$$

up to L photon losses, up to G photon gain errors, and up to D dephasing events

the quantum error-correction criteria
(the Knill-Laflamme conditions)

$$\langle 0_L|\hat{E}_i^\dagger \hat{E}_j|0_L\rangle = \langle 1_L|\hat{E}_i^\dagger \hat{E}_j|1_L\rangle,$$

and

$$\langle 0_L|\hat{E}_i^\dagger \hat{E}_j|1_L\rangle = \langle 1_L|\hat{E}_i^\dagger \hat{E}_j|0_L\rangle = 0,$$

codewords:

$$|W_{\uparrow/\downarrow}\rangle = \frac{1}{\sqrt{2^N}} \sum_{p\,\text{even/odd}}^{[0,N+1]} \sqrt{\binom{N+1}{p}}\,|p(S+1)\rangle,$$

the spacing is $S = L + G$, maximum order $N = \max\{L,G,2D\}$

break-even point:
the best uncorrectable bosonic code (0,1)
photon Fock encoding:
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Michael, Marios H., et al. "New class of quantum error-correcting codes for a bosonic mode." Physical Review X 6.3 (2016): 031006.

Qubit QEC vs binomial codes

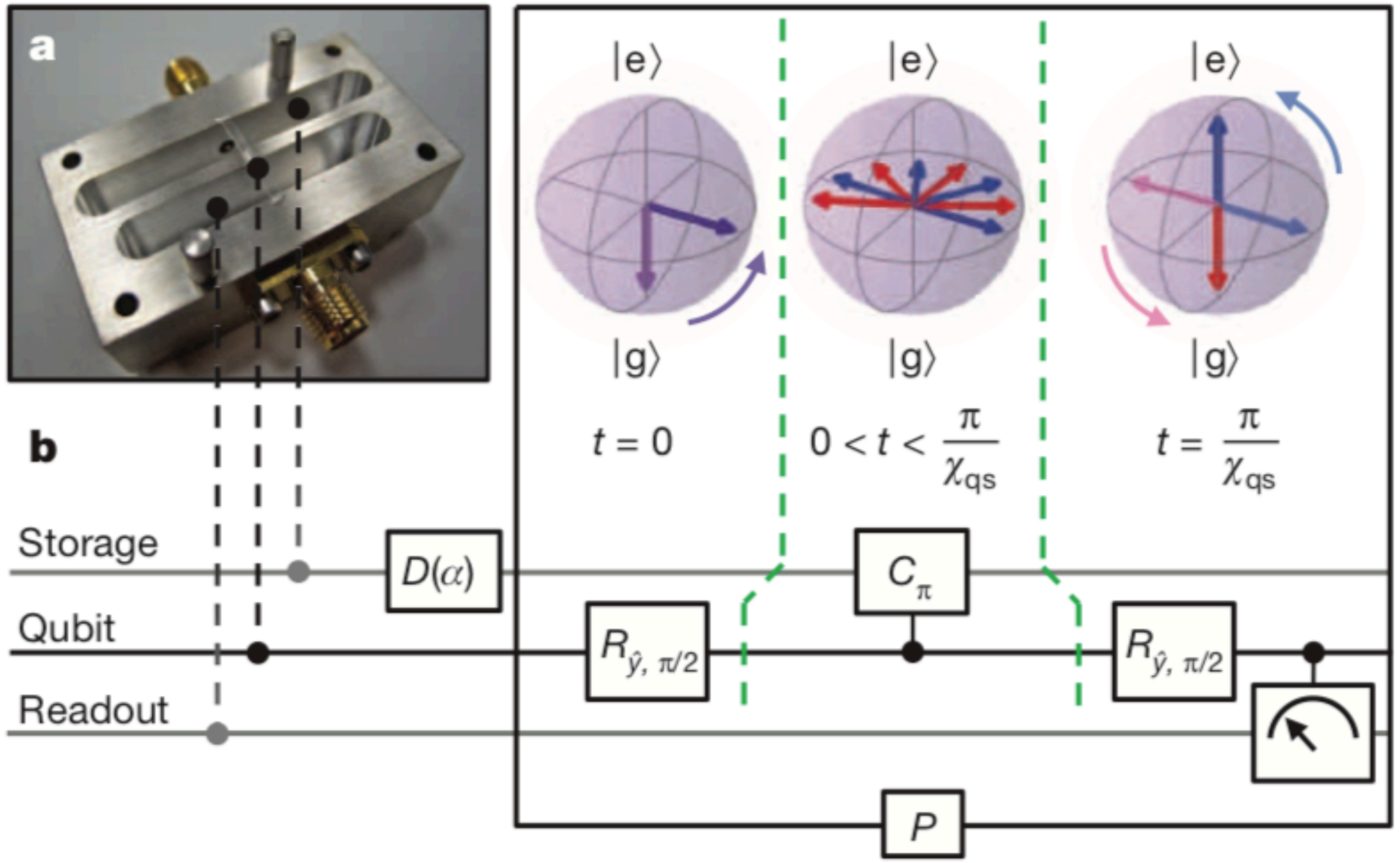
Comparison amplitude damping code

	4-qubit code	Simplest binomial code
Code word $\vert 0_\mathrm{L}\rangle$	$\frac{1}{\sqrt{2}}(\vert 0000\rangle + \vert 1111\rangle)$	$\frac{1}{\sqrt{2}}(\vert 0\rangle + \vert 4\rangle)$
Code word $\vert 1_\mathrm{L}\rangle$	$\frac{1}{\sqrt{2}}(\vert 1100\rangle + \vert 0011\rangle)$	$\vert 2\rangle$
Mean excitation number $\bar{n}$	2	2
Hilbert space dimension	$2^4 = 16$	$\{0, 1, 2, 3, 4\} = 5$
Number of correctable errors	$\{\hat{I}, \sigma_1^-, \sigma_2^-, \sigma_3^-, \sigma_4^-\} = 5$	$\{\hat{I}, a\} = 2$
Stabilizers	$\hat{S}_1 = \hat{Z}_1\hat{Z}_2,\ \hat{S}_2 = \hat{Z}_3\hat{Z}_4,\ \hat{S}_3 = \hat{X}_1\hat{X}_2\hat{X}_3\hat{X}_4$	$\hat{P} = (-1)^{\hat{n}}$
Number of Stabilizers	3	1
Approximate QEC?	Yes, 1st order in γt	Yes, 1st order in κt

Parity measurement of a photon state

QuIC introduction

Sun, Luyan, et al. "Tracking photon jumps with repeated quantum non-demolition parity measurements." Nature 511.7510 (2014): 444-448.



strong dispersive coupling

$$H/\hbar = \omega_q |e\rangle\langle e| + \left(\omega_s - \chi_{qs}|e\rangle\langle e|\right) a^\dagger a$$

Fock states associated with the qubit in the excited state acquire a phase: $\Phi = a^\dagger a \chi_{qs} t$

~ number of photons

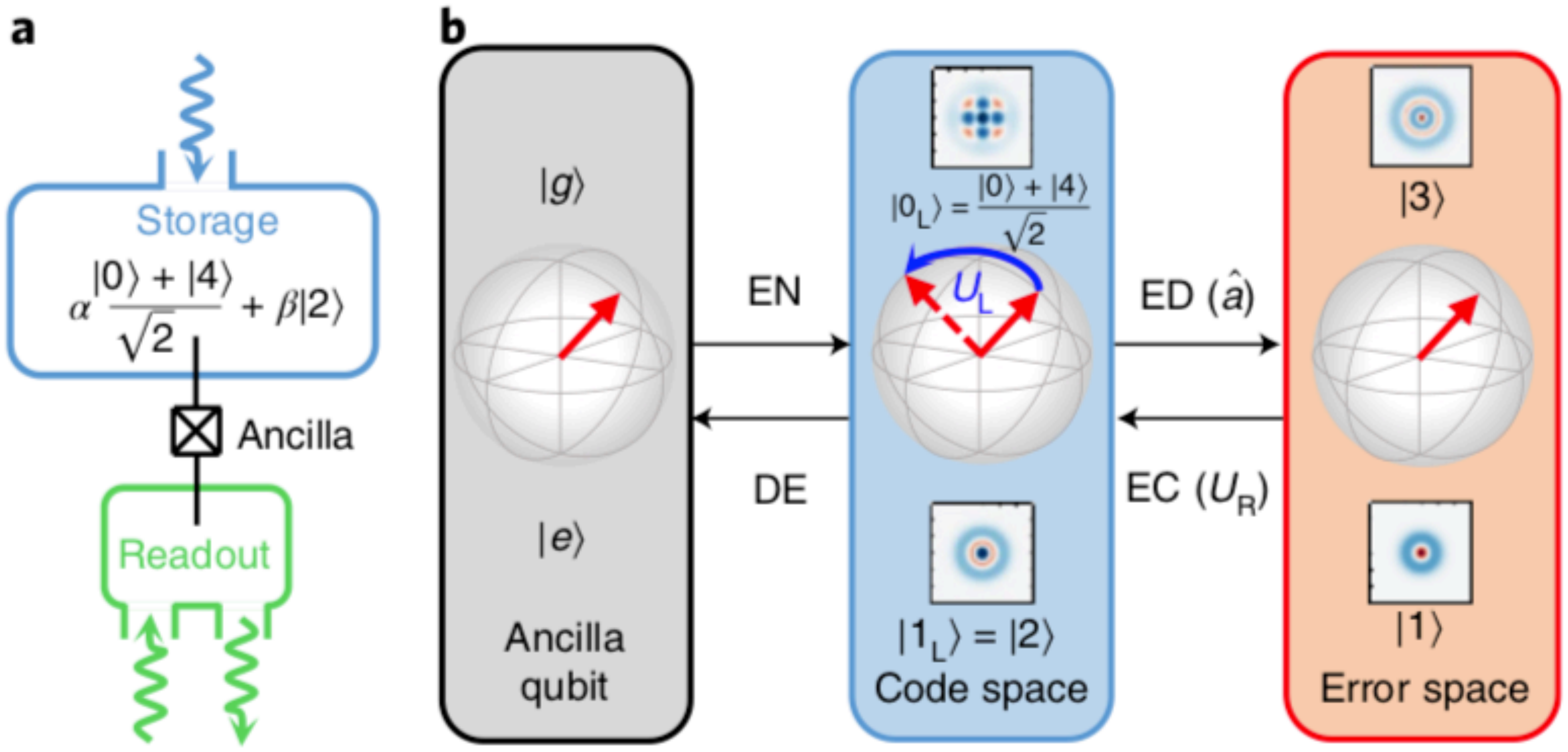
by waiting time $t = \pi/\chi_{qs}$, we realize c-phase gate

$$C_\pi = I \otimes |g\rangle\langle g| + e^{i\pi a^\dagger a} \otimes |e\rangle\langle e|$$

Simple binomial code

Experimental realization

Hu, Ling, et al. "Quantum error correction and universal gate set operation on a binomial bosonic logical qubit." Nature Physics 15.5 (2019): 503-508.



Dispersive interaction
between the ancilla and the oscillator:

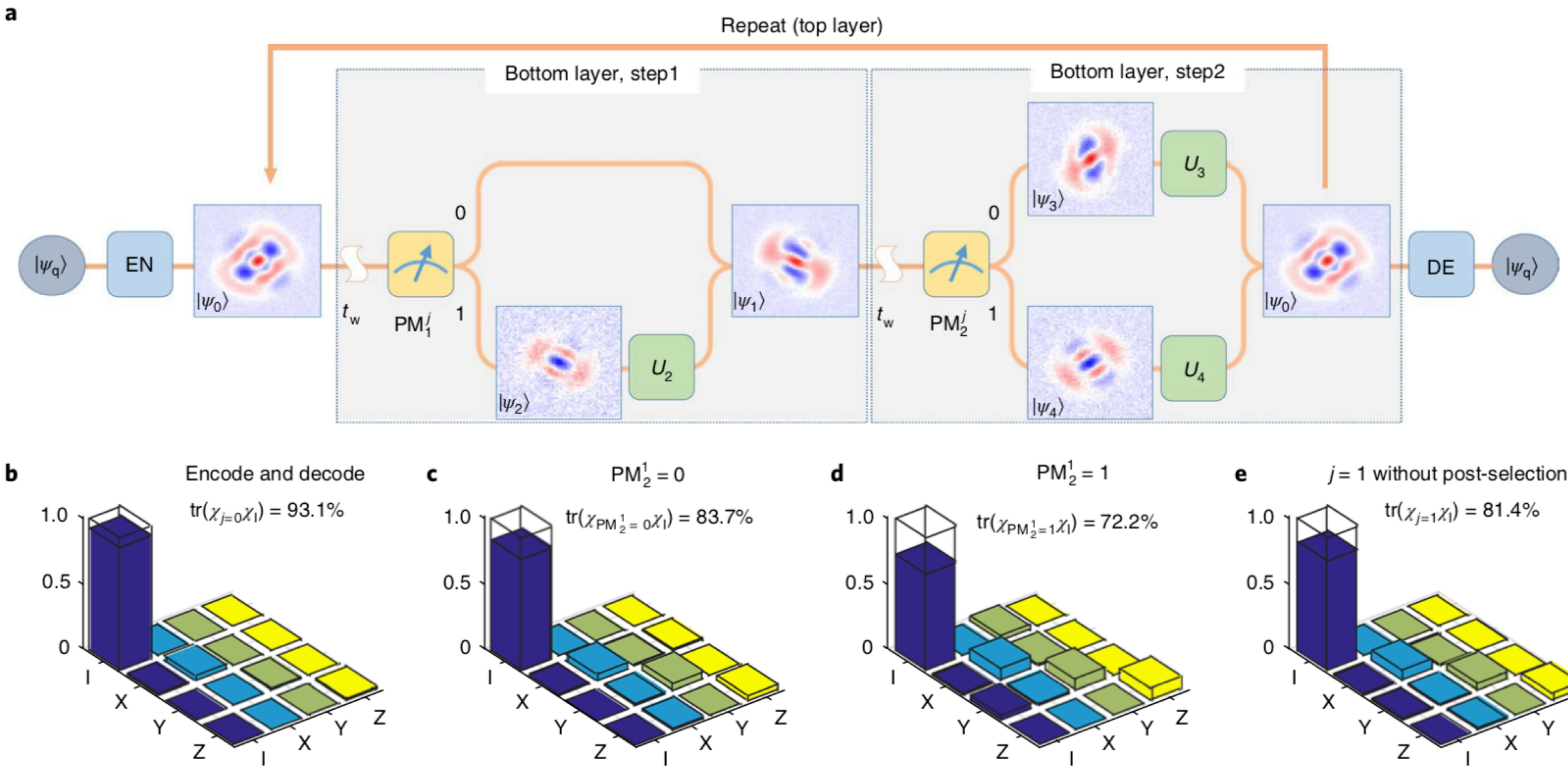
$$H_{\text{int}}=-\chi_{qs}\hat{a}^{\dagger}\hat{a}\;|e\rangle\langle e|-\frac{K}{2}\hat{a}^{\dagger 2}\hat{a}^{2}$$

interaction strength $\chi_{qs}/2\pi=1.90\,\text{MHz}$

self-Kerr coefficient $K/2\pi=4.2\,\text{kHz}$

Simple binomial code

Experimental realization / measurement protocol



Simple binomial code

Experimental realization / main results